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Reducing PAPR of OFDM signals using a tone reservation method based on ℓ_∞-norm minimization

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Abstract

Orthogonal frequency division multiplexing (OFDM) continues to be the most preferred signal-multiplexing scheme for high-speed data communication. However, OFDM signals are known to have the problem of high peak-to-average power ratio (PAPR), especially when the number of subcarriers is large, which leads to nonlinear amplification in the high power amplifier and consequently to bit-error rate degradation and out-of-band radiation. In this paper, we propose a new optimal tone reservation method for reducing high PAPR in OFDM signals in order to avoid nonlinear amplification effects. The method employs Chebyshev-norm minimization to determine peak-reduction coefficients for OFDM signal. Simulation results show that the proposed method can achieve high PAPR reduction at the expense of a small loss in data rate and a slight increase in average transmit power. For example, with 4 out of 64 subcarriers reserved for peak-reduction coefficients, which represents 6.25% data-rate loss, the method can achieve 4.06 dB of PAPR reduction with only a 0.46 dB increase in average transmit power. Similarly, when 8 subcarriers or 12.5% of the total number of subcarriers are reserved, a PAPR reduction of 5.75 dB is achieved with a paltry 0.19 dB rise in transmit power.

Keywords: High power amplifier, Orthogonal frequency division multiplexing, Peak-to-average power ratio, Tone reservation

Introduction

Recently, OFDM is the most widely used transmission technique in high data-rate applications. For example, the transmission technique is used in digital audio broadcasting (DAB), digital video broadcasting (DVB), IEEE 802.11-based wireless local area network (WLAN), IEEE 802.16-based worldwide interoperability for microwave access (WiMAX), 4th and 5th generations of mobile communication network and is a candidate technology for 6G of the same [1–5].

The extensive use of OFDM technique is due to its several important advantages; among them are high spectral efficiency, simple receiver implementation and robustness against frequency-selective fading. The high spectral efficiency comes from the use of a large number of mutually orthogonal subcarriers. The design of the receiver is simple because only single-tap equalization is needed. This is made possible by the fact that transmitted signals do not experience intersymbol interferences because of the use of sufficiently long symbol duration in conjunction with adequate time guard interval.

However, OFDM signals tend to have high PAPR that is caused by the operation of multiplexing many modulated signals. In some instances, the PAPR can reach unacceptable levels especially when large number of subcarriers is involved. A high PAPR leads to nonlinear amplification of signal by the high power amplifier (HPA) in the transmitter. This nonlinear amplification produces in-band and out-of-band radiations. The in-band radiations degrade the bit-error rate (BER), while the out-of-band radiations result in adjacent channel interferences. A simple way to avoid nonlinear amplification and the associated detrimental effects is to shift the operating point of the HPA away from the 1-dB compression point by a sufficient input back-off (IBO) depending on the expected PAPR of input signals.

However, when the HPA is provided with an IBO to force it to operate deep into the linear region, its power efficiency is reduced and thus it consumes more power. Low power efficiency requires a complex HPA design, which increases the cost of the transmitter. On the other hand, high power consumption leads to a significant reduction in lifetime of battery power in user terminals [6]. Therefore, a far much better solution is to reduce the PAPR to a suitable level before the processing of an OFDM signal in the HPA.

Recently, several PAPR reduction methods have been proposed in literature. Among them are those based on signal coding [7], clipping and filtering [8], companding [9], selective mapping [10], partial transmit sequence [11] and tone reservation [12]. PAPR reduction methods that reserve some tones for peak-cancelling signal have been found to be the most promising because they do not affect user data, and thus, the BER of the underlying system is maintained. In addition, such methods do not require transmission of side information to the receiver to aid in the recovery of user data.

The main objective of this paper is to propose an optimal tone reservation method of a good PAPR reduction performance that marginally increases the average transmission power. The method employs Chebyshev-norm approximation to find peak-reduction coefficients that yield a peak-reduction signal that closely estimates the desired signal. The proposed method can be used to reduce PAPR of signals in the current and future generations of communication networks in which OFDM technique is deployed. The performance of the proposed method is verified via simulation by the results of PAPR reduction and BER of OFDM system, which are also compared to those of other relevant and promising PAPR reduction methods.

The following is the organization of the rest of the paper. The "Related Work" section gives an overview, advantages and disadvantages of existing methods, which are relevant to this study. The section titled "PAPR in OFDM Signals" describes PAPR and its measurement. In the section "Methods", the proposed PAPR reduction algorithm is presented, while in the section headlined "Results and Discussion", simulation results and their analysis are given. Lastly, the "Conclusion" section summarises the paper.

Related work

This section gives an overview of some pertinent PAPR reduction methods and whose performances will be compared to that of the proposed method in this paper. Generally, tone reservation (TR) methods differ by the way the peak-reduction coefficients that are used to modulate the reserved tones to produce peak-reduction signal are generated. The mode of generating peak-reduction coefficients comes with its own advantages and disadvantages, with the aim being to minimize the latter to allow for practical realization.

In [13], a tone reservation method, CF-TR, based on curve fitting was proposed. The method applies curve-fitting optimization technique on a signal referred to as clipping noise to find peak-reduction coefficients for OFDM signal. Although the method has good PAPR reductions, it has to perform the computationally intensive Moore–Penrose matrix inversion in every iteration and the resulting peak-reduced transmit signal has its average power significantly increased above that of the original OFDM signal.

Another tone reservation method, LSA-TR, proposed in [14] employs least squares approximation to find peak-reduction coefficients. Although the method converges fast and the increase in average transmit power is small, its PAPR reduction performance is very poor. A TR method, IVO-TR, based on machine learning feedforward neural network and initial value optimisation is proposed in [15]. The method pre-generates and stores all possible peak-reduction signals in a pre-work table based on the training targets generated by CC-TR method [16]. At runtime, an OFDM symbol is classified and a search is done in the table for an appropriate peak-reduction signal. Although the method attempts to reduce runtime complexity, its PAPR reduction is limited by the performance of the CC-TR method whose convergence rate is affected by initial conditions. In addition, the method requires long pre-work training time to generate near optimal peak-reduction signals and increases the average transmit power.

A scaling signal-to-clipping noise ratio tone-reservation method, referred to as SSCR-TR, is proposed in [17]. The peak-reduction signal is a time-domain kernel signal obtained by scaling down the clipping noise signal by an optimal scaling vector. The LSA algorithm together with peak-regeneration constraints is employed to find the optimal scaling vector. Although the method has fast convergence rate, it is still prone to peak re-growth and the PAPR reduction performance strongly depends on the clipping ratio employed.

Another tone-reservation scheme, ELM-TR, based on online sequential extreme learning machine with a single-hidden layer feedforward neural network is proposed in [18]. The method is trained on peak-cancelling signals generated by the CC-TR scheme. Due to the use of a single hidden layer, the training time is significantly reduced. However, its PAPR reduction capability depends on the performance of the CC-TR method. Furthermore, the training of the neural network is computationally intensive and requires big storage capacity due to massive input data and numerous trainable parameters.

PAPR in OFDM signals

The OFDM signal arises from the summation of N modulated signals. Each modulated signal is basically a subcarrier signal $e^{j2\pi f_k t}$ modulated by a data symbol $X(k)$. In the discrete-time domain, the complex baseband OFDM signal in one symbol duration can be expressed in the form:

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) e^{j2\pi \frac{kn}{N}}, \quad n = 0, 1, \ldots, N-1 \tag{1}$$

The modulation symbols are obtained from binary phase-shift keying (BPSK) modulation or any M-ary quadrature amplitude modulation (M-QAM). Because (1) is similar to the inverse discrete Fourier transform (IDFT), OFDM is easily implemented using the well-known fast Fourier transform (FFT) algorithm.

Assuming a large N and that the modulated signals are statistically independent and identically distributed, then from the central limit theorem, both the real and imaginary parts of $x(n)$ have Gaussian distribution. Accordingly, the signal magnitudes $|x(n)|$ are Rayleigh-distributed, which implies that $x(n)$ can have large amplitudes well above the average value. This can in turn lead to nonlinear amplification of the large amplitudes in the HPA, thus giving rise to in-band and out-of-band radiations.

The level of peak power with reference to average power of the continuous-time OFDM signal can be estimated by the peak-to-average power ratio defined as

$$\mathrm{PAPR}\{x(n)\} = \frac{\max\limits_{0 \le n \le N-1} \left\{ |x(n)|^2 \right\}}{\mathrm{E}\{|x(n)|^2\}} \tag{2}$$

where $E\{\cdot\}$ denotes the expectation operator. In order to avoid missing the highest peak of the continuous-time signal and, therefore, wrongly estimating the PAPR using (2), the discrete-time signal $x(n)$ should be sufficiently oversampled typically by a factor greater than 4 above the Nyquist rate [19].

The level of PAPR is indicated by the complementary cumulative distribution function (CCDF) [20], which is defined as the probability that the PAPR is above a specified threshold γ, i.e.

$$\Pr\left\{ \mathrm{PAPR}\{x(n)\} > \gamma \right\} = 1 - \left(1 - e^{-\gamma}\right)^N \tag{3}$$

where Pr{.} represents the probability operator. The CCDF is normally plotted against different threshold values and this produces a curve with a waterfall-like characteristic. Since the number of subcarriers, N, in (3) is known, when considering more than one plots of CCDFs, the difference between any two thresholds at the same CCDF value measures the level of PAPR reduction. Therefore, this measurement can be used to judge how well a proposed method reduces PAPR.

Methods

The proposed method utilises the concept of tone reservation [21] in which a smaller number of OFDM subcarriers, which were previously intended for the transportation of user data, are reserved to carry PAPR reduction coefficients. The reserved subcarriers are referred to as peak-reduction tones. Because the reserved subcarriers do not carry user data, a data-rate loss expressed as

$$R_f = \frac{L}{N} \tag{4}$$

is expected in a commrununication system employing a PAPR reduction method based on the tone reservation concept. In (4), L and N denote the number of reserved subcarriers and total number of subcarriers in one OFDM symbol, respectively.

In order to minimize the data-rate loss in (4), the number of reserved subcarriers should be set much smaller than the total numbers of subcarriers, i.e. $L \ll N$. In addition, to avoid distorting the user data due to the introduction of peak-reduction coefficients, the peak reduction tones and the data-bearing subcarriers are made to occupy two disjoint frequency subspaces in every OFDM symbol. Thus, in the reserved subcarrier positions, there are no modulating data symbols, i.e. they are set to zero. Likewise, in the locations allocated for subcarriers for user data, the peak-reduction coefficients are set to zero. At the receiver, the disjoint frequency subspaces allow the transmitted symbols to be recovered from the FFT output without distortion by considering only the locations of data-bearing subcarriers.

The tone reservation concept that has just been described is illustrated in Fig. 1, where $X(k)$ and $C(k)$ are the modulating data symbols and peak-reduction coefficients, respectively. As shown in the figure, after the inverse FFT (IFFT) operation, the resulting combined signal $\mathbf{s} = \mathbf{x} - \mathbf{c}$ has a reduced peak amplitude and hence lower PAPR than the original time signal $\mathbf{x}$. After reduction in PAPR, the combined signal is converted into an analogue signal by a digital-to-analogue converter (DAC) then up-converted to radio frequency f_c before being passed to HPA for power amplification.

The generation of a low PAPR transmit signal $s(n)$ using the tone reservation concept can be described by the equation

$$\begin{aligned} s(n) &= x(n) - c(n) \\ &= x(n) - \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} C(k) e^{j2\pi \frac{kn}{N}} \end{aligned} \tag{5}$$

or in matrix notation as

$$\mathbf{s} = \mathbf{x} - Q\mathbf{C} \tag{6}$$

Here, $Q \in \mathbb{C}^{N \times N}$ is the IDFT matrix and contains the elements given by $(1/\sqrt{N})\exp\left(j2\pi kn/N\right)$, $\mathbf{s} = [s(0), s(1), \ldots, s(N-1)]^{\mathrm{T}}$ is the peak-reduced signal vector, $\mathbf{x} = [x(0), x(1), \ldots, x(N-1)]^{\mathrm{T}}$ contains samples of original OFDM signal and $\mathbf{C} = [C(0), C(1), \ldots, C(N-1)]^{\mathrm{T}}$ is a frequency-domain vector of the peak-reduction coefficients.

If we let $\hat{\mathbf{C}} \in \mathbb{C}^{L}$ denote the vector containing the L nonzero elements of $\mathbf{C}$, the peak-reduction signal $\mathbf{c}$ can be expressed as

$$\mathbf{c} = \hat{Q}\hat{\mathbf{C}} \tag{7}$$

where the IFFT submatrix $\hat{Q} \in \mathbb{C}^{N \times L}$ is made up of L columns of Q corresponding to the locations of the reserved subcarriers.

Proposed method

Ideally, the peak-reduction signal can be considered to consist of samples of the difference signal between the original OFDM samples and the clipped

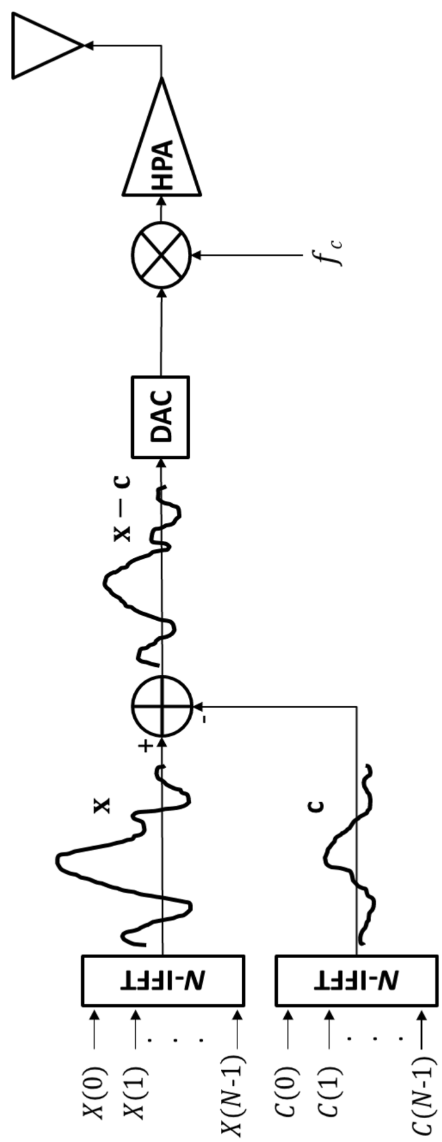

Fig. 1 Tone reservation concept

version. The clipped signal is derived by clipping the OFDM signal at a threshold x_{th}. Analytically, the desired peak-reduction signal can be expressed as a vector $\mathbf{d} = [d(0), d(1), \ldots, d(N-1)]^{\text{T}}$ with components given by

$$d(n) = \begin{cases} \frac{x(n)}{|x(n)|}(|x(n)| - x_{\text{th}}), & |x(n)| > x_{\text{th}} \\ 0, & |x(n)| \leq x_{\text{th}} \end{cases} \tag{8}$$

The saturation point of the HPA can be used to determine the clipping threshold. Given the maximum PAPR allowed in an OFDM-based communication system, the clipping threshold can be found from (2) as follows:

$$x_{\text{th}} = \sqrt{\text{PAPR}_{\text{max}} E\{|x(n)|^2\}} \tag{9}$$

For effective reduction in PAPR, the actual peak-reduction signal should be close or equal to the desired signal. In other words, the residual error

$$\mathbf{r} = \hat{Q}\hat{\mathbf{C}} - \mathbf{d} \tag{10}$$

between the two signals should be as small as possible if not equal to zero. This can be achieved through the minimization of the residual error using an appropriate norm to measure the error level. Noting that the highest peak in $\mathbf{d}$ is the main cause of high PAPR in signal $\mathbf{x}$, it is preferable to minimize the Chebyshev (ℓ_∞) norm of the residual error in order to ensure that the largest error magnitude is minimized.

From the foregoing discussion, the problem of minimizing the residual error can be formulated as the following Chebyshev approximation problem [22]:

$$\text{minimize} \quad ||\hat{Q}\hat{\mathbf{C}} - \mathbf{d}||_\infty \tag{11}$$

where $||.||_\infty$ denotes the ℓ_∞-norm.

The Chebyshev approximation problem (11) has no closed form solution but can be solved after casting it into the following linear program:

$$\begin{array}{ll} \text{minimize} & t \\ \text{subject to} & \hat{Q}_n^{\text{T}}\hat{\mathbf{C}} - t \leq d(n) \\ & -\hat{Q}_n^{\text{T}}\hat{\mathbf{C}} - t \leq -d(n) \end{array} \tag{12}$$

in which $t \in \mathbb{R}$ and $\hat{\mathbf{C}} \in \mathbb{C}^L$ are the optimization variables and $\hat{Q}_n \in \mathbb{C}^L$ and $d(n) \in \mathbb{C}$, for $n = 0, 1, 2, \ldots, N-1$, are the problem parameters. Note that, $\hat{Q}_n$ is a column vector equal to the transpose of the nth row of matrix $\hat{Q}$.

After solving (12), the time-domain peak-reduction signal and the transmit signal are obtained using (7) and (6), respectively. The main steps of the proposed algorithm are listed in Table 1.

The effectiveness of the proposed method in terms of peak-power reduction can be measured by comparing the level of the maximum power of the peak-reduced signal to that of the average power of the original OFDM signal. In order to do such a comparison, the peak-to-average power ratio of the peak-reduced signal is defined as follows:

Table 1 Proposed TR algorithm

i	Set number of subcarriers N, allowed data rate loss R_f and maximum allowed $\mathrm{PAPR}_{\max}$
ii	Generate OFDM signal $\mathbf{x}$ and calculate PAPR
iii	If calculated PAPR $<$ $\mathrm{PAPR}_{\max}$, transmit $\mathbf{x}$ and terminate program, else got to step (iv)
iv	Calculate clipping threshold $x_{\mathrm{th}} = \sqrt{\mathrm{PAPR}_{\max} E\left\{\lvert x(n)\rvert^2\right\}}$
v	Generate desired peak-reduction signal $\mathbf{d}$
vi	Generate IFFT submatrix $\hat{Q} \in \mathbb{C}^{N\times L}$
vii	Use interior-point method to minimize $\lvert\lvert\hat{Q}\hat{\mathbf{C}} - \mathbf{d}\rvert\rvert_\infty$ and solve for PRCs vector $\hat{\mathbf{C}}$
viii	Compute peak-reduction vector $\mathbf{c} = \hat{Q}\hat{\mathbf{C}}$
ix	Compute $\mathbf{s} = \mathbf{x} - \mathbf{c}$ and transmit
x	End

$$\mathrm{PAPR}\{s(n)\} = \frac{\max\limits_{0\leq n\leq N-1}\left\{\lvert x(n) - c(n)\rvert^2\right\}}{E\{\lvert x(n)\rvert^2\}} \tag{13}$$

One drawback of the just proposed method is that it increases the transmit power, i.e. the average power of signal **s** will be higher than the average power of signal **x**—a problem that is inherent to all methods based on the tone reservation concept. However, because the desired peak-reduction signal has most of the components equal to zero, the peak-reduction signal resulting from the Chebyshev approximation is expected to have most of its samples very small, close zero, and thus the increase in the average power will be small.

Computational complexity

Depending on the size of the problem in (12), it can be solved using one of the three linear programming algorithms [23], namely interior-point, active-set and simplex algorithms to find the peak-reduction coefficients. The proposed method will employ the interior-point method to solve (12). Since there is no simple analytical formula for the solution to a linear program, the required number of arithmetic operations cannot exactly be established.

However, in practice, the interior-point method is known to have a complexity $O\left(NL^2\right)$, where N and L are number of rows and columns of matrix $\hat{Q}$, respectively [22]. Additionally, since $\hat{Q}$ is a submatrix of the well-structured IDFT matrix, the linear program in (12) can be solved with complexity $O\left(N\log_2 N\right)$ [24].

The complexity of the proposed method and those in the related work section are listed in Table 2. In the table, I, I_{sscr}, I_{cf} and I_{ls} denote the respective number of iterations required to find the peak-reduction coefficients at runtime for the proposed method, SSCR-TR, CF-TR and LSA-TR methods. For the ELM-TR and IVO-TR methods, N_{s}, N_{i}, N_1 and N_{o} denote the size of the training-data set and the number of neurons in the input, hidden and output layers, respectively.

Results and discussion

The proposed method was employed to reduce PAPR in OFDM systems with $N = 64$ subcarriers. The simulation parameters, which are listed in Table 3, were purposely chosen to help ascertain the performance of the method in terms of PAPR reduction and BER degradation and to allow for comparison with other methods. The problem in (12)

Table 2 Computational complexity comparison

Method	Training complexity	Runtime complexity
Proposed method	None	$I \times O\left(N\log_2 N\right)$
ELM-TR	$O(N_iN_1 + N_sN_1N_o)$	$O(N_iN_1 + N_1N_o)$
SSCR-TR	None	$I_{sscr} \times O\left(N + N\log_2(N)\right)$
IVO-TR	$N_s \times O(N_iN_1 + N_1N_o + MN_o)$	$O(N_iN_1 + N_1N_o)$
CF-TR	None	$I_{cf} \times O\left(N\log_2 N\right)$
LSA-TR	None	$2I_{ls} \times O\left(N\log_2 N\right)$

Table 3 Simulation parameters

Total subcarriers, N	64
Subcarrier modulation	QPSK
Number of OFDM symbols	10^4
Oversampling factor	4
Number of reserved subcarriers, L	4, 8
Power amplifier model	Rapp model, $p=2$, IBO = 8 dB

was first re-formulated in MATLAB to take into account the real and imaginary parts of the inequality constrains. The interior-point method was then employed to solve the linear program for the peak-reduction coefficients.

To have a good estimate of the continuous-time PAPR, all the discrete-time signals were oversampled by a factor of 4. In addition, the Rapp's model of HPA [25] was used with the smoothness parameter p set at 2 and the IBO at 8 dB, which is approximately 1 dB above the PAPR of peak-reduced signals at the CCDF $= 10^{-3}$. This IBO setting ensures that the percentage of signal amplitudes clipped by HPA is less than 1%. Additionally, since the type of subcarrier modulation does not affect the level of PAPR reduction, only the QPSK modulation was used during the simulations.

In Fig. 2, the PAPR reduction performance of the proposed method is shown for two cases of 4 and 8 reserved subcarriers out of the 64 subcarriers. The two cases give a data-rate loss of 6.25% and 12.5%, respectively. The PAPR reduction at CCDF $= 10^{-3}$ is 4.06 dB and 5.75 dB for 4 and 8 reserved subcarriers, respectively. This shows that the proposed method can achieve high PAPR reductions with only a small percentage of the total number of subcarriers reserved for peak-reduction coefficients. It can also be observed that the reduction in PAPR increases with the number of reserved subcarriers. However, since the PAPR reduction is at the expense of a data-rate loss due to the reserved subcarriers, a compromise between the two is necessary depending on the requirements of the communication system.

The PAPR reduction for the case of 4 reserved subcarriers was used to compare the performance of the proposed method to ELM-TR, SSCR-TR, IVO-TR, CF-TR and LSA-TR methods. For the IVO-TR and ELM-TR methods, $N_O = 400$, $N_1 = 1000$, $N_S = 10^5$ and the size of the test-data set is 10^4. In addition, the training target is 100 iterations of CC-TR method. The number of iterations for the CF-TR, LSA-TR and SSCR-TR methods is 2, 3 and 5, respectively.

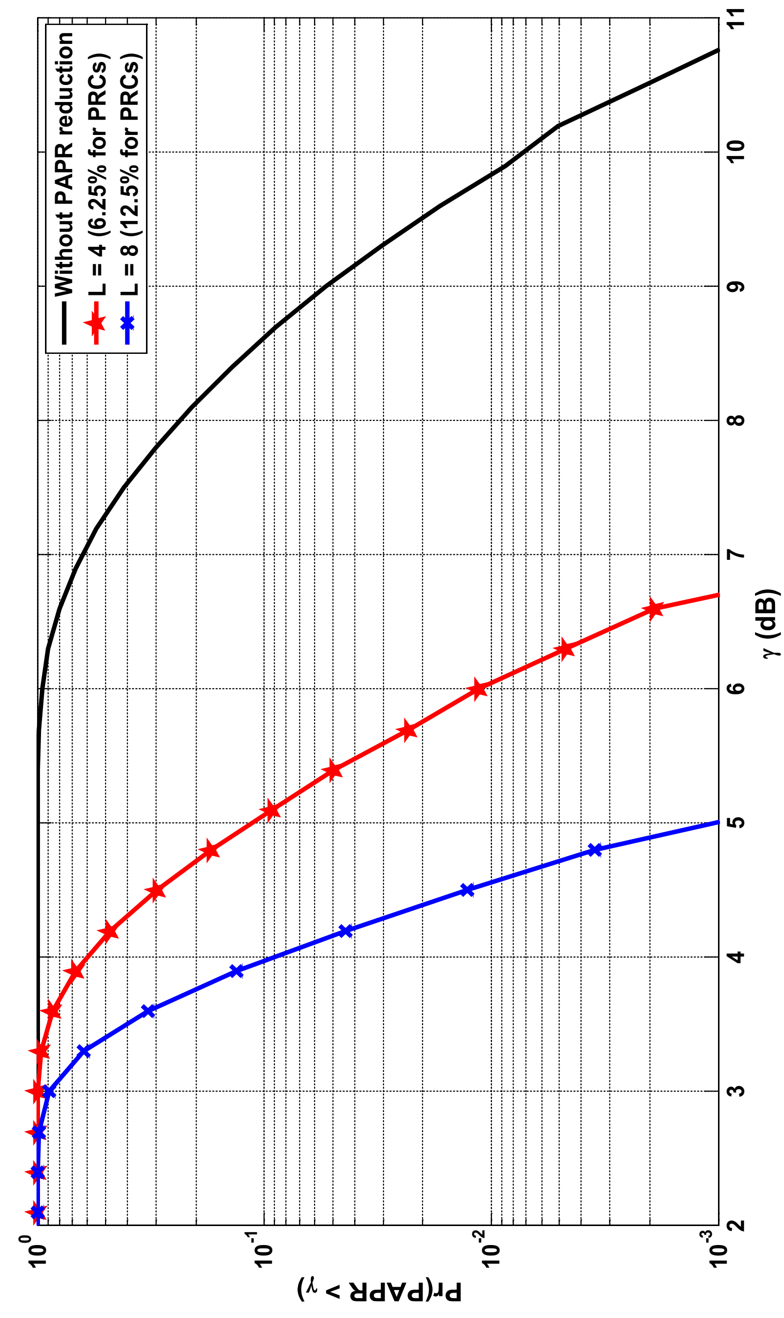

Fig. 2 PAPR reduction with 4 and 8 reserved subcarriers

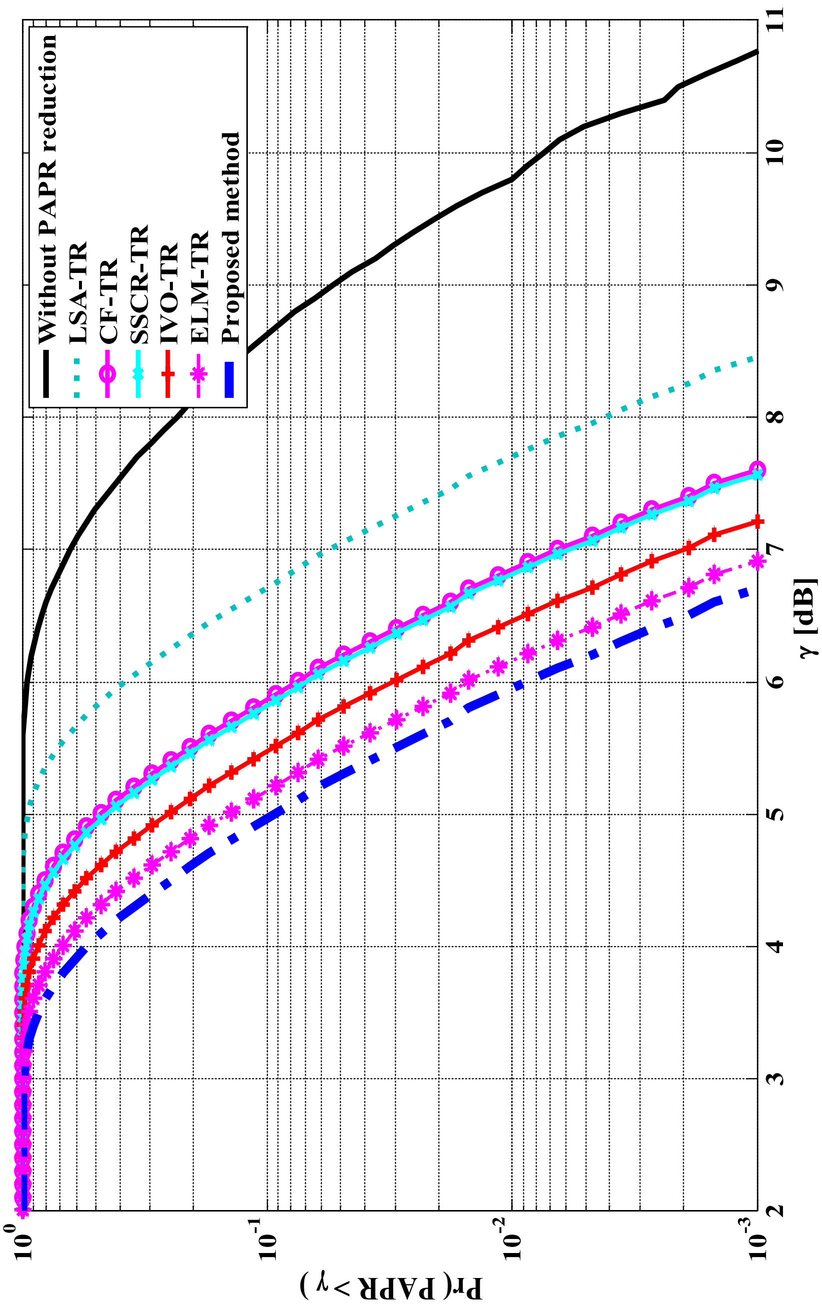

Fig. 3 PAPR reduction performance comparison

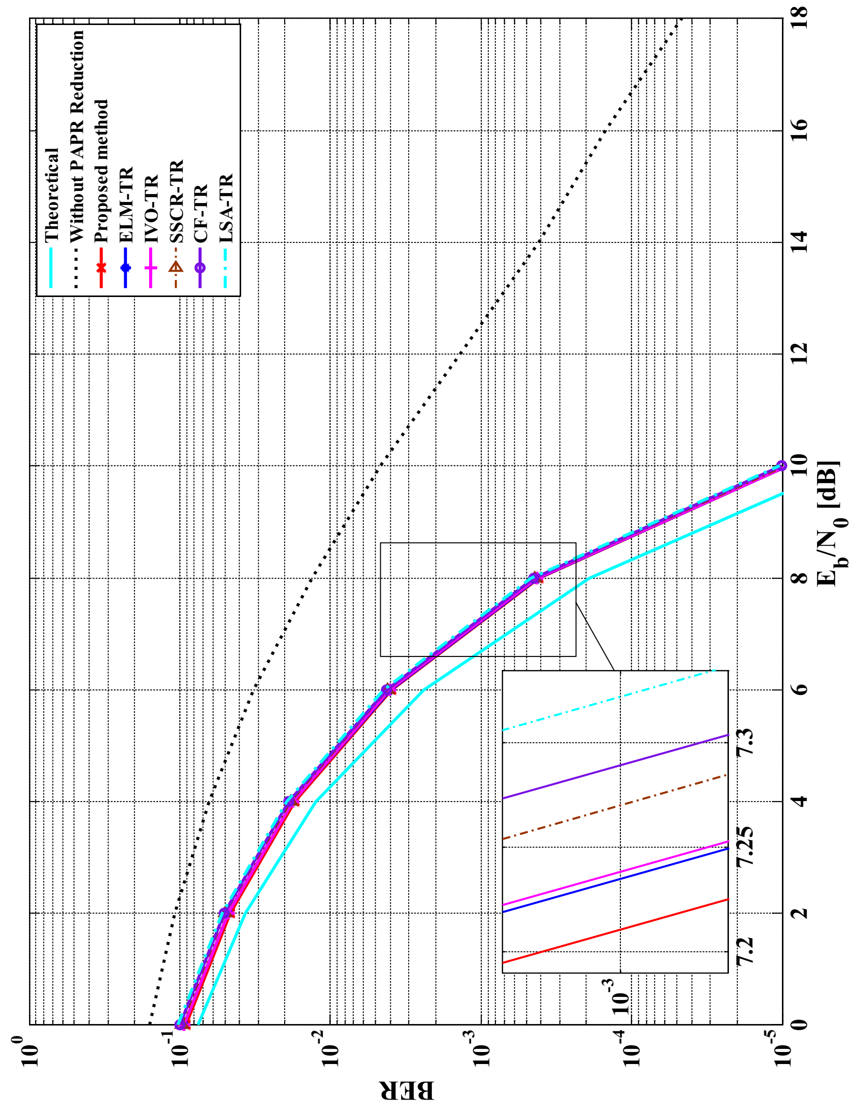

Fig. 4 BER performances over AWGN channel

Table 4 PAPR reduction and average power increase

Method	PAPR reduction [dB]	Power increase [dB]
Proposed LP-TR	4.06	0.46
ELM-TR	3.85	0.59
SSCR-TR	3.19	0.25
IVO-TR	3.55	0.57
CF-TR	3.16	0.84
LSA-TR	2.31	0.30

Table 5 Required E_b/N_o at BER $= 10^{-3}$

Method	E_b/N_o (dB)
Proposed method	7.21
ELM-TR	7.23
SSCR-TR	7.27
IVO-TR	7.24
CF-TR	7.29
LSA-TR	7.32

The CCDF curves for the six methods are depicted in Fig. 3, and the results for PAPR reductions at CCDF $= 10^{-3}$ and the average power increase are summarized in Table 4. From these results, it can be observed that the proposed method has better PAPR reduction performance than the rest. At the CCDF $= 10^{-3}$, the proposed method exhibits a higher PAPR reduction than the ELM-TR, SSCR-TR, IVO-TR, CF-TR and LSA-TR method by 0.21, 0.87, 0.51, 0.90 and 1.75 dB, respectively.

For the BER performance, the results for the six methods are given in Fig. 4. The BER performances are for the cases of transmission of amplified peak-reduced signals over additive white Gaussian noise (AWGN) channels. The curve labelled *Theoretical* gives the lower limit or the best-expected performance as it corresponds to the performance given by the BER formula of the QPSK modulation. The curve labelled *Without PAPR Reduction* is for the case when the OFDM signals were amplified through the HPA with the IBO = 0 dB and therefore is the worst expected BER performance.

The required SNR per bit, i.e. E_b/N_o, at BER $= 10^{-3}$ for the methods is presented in Table 5. As it is expected of methods based on tone-reservation concept, all the six methods have approximately the same BER performance. However, due to the setting of the IBO and the level of PAPR reduction, the proposed LP-TR method has a slightly better BER performance than ELM-TR, SSCR-TR, IVO-TR, CF-TR and LSA-TR method by 0.02, 0.06, 0.03, 0.08 and 0.11 dB, respectively.

Conclusions

In this work, we have proposed a new optimal tone reservation method for reducing PAPR of OFDM signals. The method first generates a desired peak-reduction signal, and then, using linear programming of the Chebyshev approximation problem, it designs the actual peak-reduction signal, while utilising only a small number of reserved subcarriers for peak-reduction coefficients.

With a small number of reserved subcarriers, the proposed method achieves significant PAPR reductions, e.g. with 4 and 8 reserved subcarriers out of a total of 64, 4.06 and 5.75 dB of PAPR reductions are attained, respectively. In addition, the method only causes only a small increase in transmit power, e.g. for the case of 4 reserved subcarriers, the power increase is 0.46 dB. Additionally, the method does not affect the BER of the underlying OFDM system.

In comparison with five other methods, namely ELM-TR, SSCR-TR, IVO-TR, CF-TR and LSA-TR method, the proposed method has better PAPR reduction performance. At CCDF $=10^{-3}$ for the case of 4 reserved subcarriers out of 64, the proposed method achieves 0.21, 0.87, 0.51, 0.90 and 1.75 dB of PAPR reduction above the ELM-TR, SSCR-TR, IVO-TR, CF-TR and LSA-TR method, respectively.

In future work, the proposed method can be employed to reduce PAPR in an OFDM system employing adaptive modulation and coding during one symbol duration. Additionally, the peak-reduction signals generated by the proposed method can be used as training targets for a PAPR reduction method based on machine learning. Another future research is to develop a faster algorithm than the interior-point algorithm to solve the formulated Chebyshev approximation problem in this paper and thereby reduce convergence time and computational complexity.

Abbreviations

AWGN Additive white Gaussian noise
BER Bit-error rate
SNR Signal-to-noise ratio
CCDF Complementary cumulative distribution function (CCDF)
CF Curve fitting
HPA High power amplifier
IBO Input back-off
IVO Initial value optimization
OFDM Orthogonal frequency division multiplexing
LSA Least squares approximation
PAPR Peak-to-average power ratio
QAM Quadrature amplitude modulation
QPSK Quadrature phase-shift keying
TR Tone reservation

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Author contributions
SK conducted the research, analysed the data and wrote the paper. EM and GK reviewed and corrected the paper. All authors approved the final version of the paper.

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Availability of data and materials
The data used and/or analysed during the current study are available from the corresponding author on reasonable request.

Declarations

Competing interests
The authors declare that they have no competing interests.

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