Babu *et al.*
Journal of Electrical Systems and Inf Technol (2023) 10:42
https://doi.org/10.1186/s43067-023-00110-4

Journal of Electrical Systems and Information Technology

RESEARCH **Open Access**

Allocation of synchronized phasor measurement units for power grid observability using advanced binary accelerated particle swarm optimization approach

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Abstract

Large-scale power grid observability is still a challenge because of deteriorating infrastructure and the incorporation of renewable energy sources. A smart grid that makes use of cutting-edge technology, such as a phasor measurement unit (PMU), is an excellent option for monitoring and bringing networks up to speed with the latest information. Latterly, the considerable investment required for the deployment locations has slowed down the adoption of PMU. Therefore, because PMUs are expensive, it is necessary to deploy them in the best possible places on large-scale power grids. The most significant share of optimal PMU placement problems (OPPP) is defined as 0–1 knapsack problems. Considering this, the development of an effective optimization technique that can handle difficulties has emerged as an appealing topic in recent years. In this paper, a meta-heuristic algorithm based on the binary particle swarm algorithm (BPSO), a binary accelerated particle swarm optimization (BAPSO), is offered for solving OPPP. Since earlier research has shown that BPSO is likely to stick to local optima, the majority of them evaluated their suggested technique using small-scale test systems. The technique that has been suggested searches for the optimal solution by employing two topologies—one global and one local—that are analogous to BPSO. This work determines the optimal PMU position for a large network in a reasonable amount of time by fine-tuning the acceleration factor. Additionally, in order to employ fewer PMUs, an integration strategy was put into place for the radial buses. The OPPP solutions are provided by the suggested method within a reasonable period with prior solutions published in reliable publications, according to computational findings.

Keywords: Binary accelerated particle swarm optimization, Binary particle swarm optimization, Optimal PMU placement problem, Phasor measurement unit, Redundancy measurement

Introduction

The power networks of today are being run under difficult conditions in order to supply the rapidly growing demand for electrical resources and to keep the commercial activity going in the midst of a very dynamic, deregulated market. Therefore, power grid monitoring, preservation, and control become increasingly important for enhanced systems operation, maintenance, planning, and energy trading. As a result, PMU has evolved as a valuable piece of equipment for measuring phasors of voltage and current, which are synchronized with signals collected using GPS technology. They can enhance operations such as bad data detection, corrective action schemes, state estimation, stability control, and disturbance monitoring. When it comes to installing PMU on the electrical grid, one of the most important issues that have to be taken into account is the expense of doing so. It is, therefore, of the utmost importance to determine the optimum position of the PMU where reliability is maintained while minimizing the costs involved. Recently, many methods to analyze the OPPP employing various sets of optimization algorithms have been presented. Deterministic and stochastic algorithms are two categories that describe optimization strategies that can be used to solve the PMU placement problem.

The integer linear programming (ILP)-based formulation to evaluate the OPPP was initially suggested in [1]. This formulation, in which linear constraints are established based on a binary bus-to-bus connection matrix, evaluates the observability of power networks considerably more simply and straightforwardly. An ILP method was suggested in [2]; this method took traditional measurement as well as zero injection bus (ZIB) into consideration. The use of a permutation matrix is included in the suggested method, which helps reduce the nonlinear limitations. There is also an explanation of the idea of partial observability in [3]. In addition, a malfunctioning of PMU was incorporated into the strategy that was presented. The bus observability index (BOI) and the system observability redundancy index (SORI) were described so that the optimal PMU employment set could be obtained.

To address the effective solutions specified for the OPPP while taking into consideration the impact of ZIB, such as line and PMU outages, a mixed ILP (MILP) is presented in [4] and [5]. A technique that is based on integer programming and genetic algorithms (GA) was developed by [6] to install PMU in order to obtain full monitorability of the power network. A combination of GA with a simulated annealing strategy was offered by Kerdchuen and Ongsakul in [7] as a way of obtaining a solution for the OPPP. In [8], researchers investigated a unique cellular GA-based approach for OPPP that takes into consideration the availability of channel capacity as well as single-line loss. Ahmadi et al. [9] recommended using conventional BPSO to decide on the OPPP with and without ZIBs. The measurement of redundancy is presented as a method for ranking the solutions.

In the research conducted by Chakrabarti et al. [10], an enhanced particle swarm optimization (EPSO) for power grids, as described by Valle et al. [11], was applied to the OPPP. Further velocity update rules are implemented by EPSO if the particles cannot identify a viable solution. Similar to the study conducted by Chakrabarti et al. [10], the authors of [12] suggested a novel velocity update equation to locate the OPPP using BPSO. In addition to the velocity update equation, the authors created additional observability techniques for ZIB, a PMU, and a line failure. In reference [13], the authors introduced the exponential BPSO as a novel way of controlling the inertia mass of BPSO. The authors assert that it improves the searchability of the method. Wang et al. [14] presented a hybrid technique for the OPPP that combines simulated annealing and BPSO. In order to place PMUs in power distribution systems optimally, a tri-objective strategy has been presented in [15]. Its goals are to reduce the number of PMU channels, state estimation uncertainty, and sensitivity to line parameter tolerances. Observability propagation depth and probabilistic observability are taken into account in [16] to improve formulation for the best placement of PMUs in power grids.

A two-stage approach to optimize the placement of PMUs was proposed in [17, 18] to achieve complete system visibility while minimizing cost, taking into account objectives such as cost minimization, redundancy, and efficiency maximization, as well as constraints such as zero injection buses, single PMU failure, single-line outage, and flow measurements. Article [19] addresses the issue of incomplete observability under single PMU loss ($N-1$) contingencies and proposes an enhanced two-archive algorithm and a fuzzy decision-making method for PMU placement optimization. In ref. [20], author has presented a BPSO technique for the optimal allocation of PMUs in connected power networks, demonstrating its effectiveness and superiority compared to other methods through testing on various test systems. In addition, a technique for integration is not used for the radial buses; instead, the approach entails taking into consideration as well as ignoring ZIBs.

In this article, author has proposed a meta-heuristic algorithm, based on BPSO, a BAPSO, to solve the OPPP in large-scale power grids, aiming to find the best locations for deploying expensive PMUs and achieve grid observability while considering the challenges posed by deteriorating infrastructure and cost constraints. The algorithm combines global and local search topologies and fine-tunes the acceleration factor to efficiently determine the optimal PMU positions, and it also incorporates an integration strategy for radial buses to reduce the number of PMUs required, providing solutions within a reasonable time frame compared to the previous research. As BAPSO is a meta-heuristic algorithm, it is expected to generate multiple PMU placement sets and to determine the quality of each set with the same number of PMUs, the one with the highest SORI value is chosen as the optimal result. The PMU placement set with higher measurement redundancy is considered better than the one with lower measurement redundancy. BAPSO is proposed to determine the minimum number and optimal locations of PMUs for complete monitoring of the power grid, taking into account factors such as normal operation and zero injection measurements.

Method used for the optimal PMU placement problem

In general, the primary goal of the OPPP is to obtain the fewest number of PMUs necessary, along with the location of those PMUs, to ensure full observability of the power grid. As a result, the following is the model for the generalized objective function that is used for the identification of the OPPP in this work [21]:

$$min \sum_{i=1}^{n} c_i.y_i = \sum_{i=1}^{n} y_i.(c_i = 1) \tag{1}$$

$$\begin{aligned} &\text{subject to :} \\ &HA_{\mathrm{PMU}}Y \geq B_{\mathrm{PMU}} \end{aligned} \tag{2}$$

$$Y = [y_1\, y_2\, y_3 \ldots y_n]^T \tag{3}$$

$$y_i \in \{0, 1\} \tag{4}$$

where n is the number of buses, c_i is the vector of PMU price coefficients, Y is the binary design variable vector having components y_i which decide the feasibility of PMUs on ith bus, and H and B_{PMU} are interpreted as the transformation matrix that may be modified according to the contingency cases. $A_{PMU} = [A_{i,k}]_{n\times n}$ is the binary connectivity matrix that explains the bus-to-bus connection whose entries are shown in Eq. (5).

$$A_{i,k} = \begin{cases} 1, \text{ if bus } i \text{ and bus } k \text{ are linked} \\ 1, \quad\quad\quad \text{if } i = k \\ 0, \quad\quad\quad \text{otherwise} \end{cases} \tag{5}$$

Y provides the decision for the placement of PMU as given in Eq. (6).

$$y_i = \begin{cases} 1, \text{ if PMU is set - up at } ith \text{ bus} \\ 0, \quad\quad\quad \text{otherwise} \end{cases} \tag{6}$$

$B_{\text{PMU}} = [B]_{n\times 1}$ is the column vector that signifies the redundancy, that is, essential for the specific case.

$$B_{\text{PMU}} = [1\,1\,1\,..\,1]^T \tag{7}$$

Radial BUS

It is noted that installing PMU on a bus that is linked to more than one neighboring bus would have greater coverage of the connected power grid relative to the bus that has very few adjacent buses, in particular, the radial bus network [22]. Hence, if the PMU-equipped bus is radial, the PMU can only monitor two buses—the radial bus and its neighbor. Radial buses are excluded from prospective OPPP solutions since their PMU setup will measure the voltage phasors on that bus and one associated bus.

Modeling of ZIB

The consideration of ZIB may benefit in further reducing the PMU numbers necessary to achieve maximal observability of the power grid. Several methods for coping with ZIBs have been suggested in the previous research. The bus integration approach is one of the strategies that have been established to cope with the characteristics of the ZIB [22]. The bus integration strategy requires an integration process between the ZIB and one of the neighboring buses. As a consequence of this, during the process of integration, the limits placed on both buses may be combined into a single constraint. As a result, the number of constraints that need to be satisfied to guarantee that the installed PMUs will observe each bus will be reduced. It is believed that if all observable buses except for the unobservable one are interconnected to the ZIB, then the unobservable bus can be construed as being observable. Because of this, the integration of the bus shows that if it is measurable, the bus that was picked to be integrated will also be observable.

The 14-bus system of IEEE is taken into consideration to comprehend the bus integrating strategy. This system is illustrated in Fig. 1, and it is important to note that bus "7" is a ZIB, and it is coupled with bus "4, 8," and "9." To identify a candidate bus to integrate with the ZIB, the following process may be utilized: (i) randomly integrate the ZIB with one of the buses that are near it. In this example, bus 7 is integrated with one of its neighbors. As an illustration, bus 7 and bus 9 are combined into one, (ii) integrate the

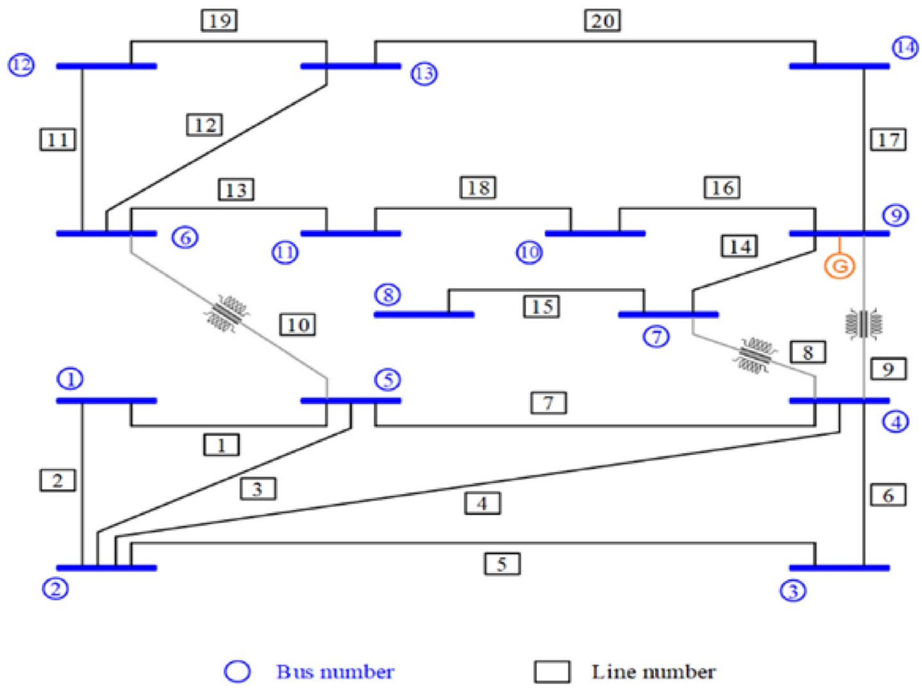

Fig. 1 Single-line diagram of IEEE 14-bus systems

ZIB with one of the surrounding buses that have the fewest number of buses attached to it. Using this technique, bus 7 is integrated with bus 8, which only has one bus connected to it; therefore, the total number of buses linked to bus 8 is reduced to one, and (iii) integrate the ZIB with its adjacent buses that have a higher number of buses linked to it—in contrast with the plan that was shown previously, bus number 7 is interconnected with bus number 4, which has a total of five buses that are connected to it. Bus number 9 only has a total of four buses that are connected to it.

When dealing with the presence of ZIB, a bus integrating approach may be used to establish the bare minimum number of PMUs that are required; despite this, there are a few drawbacks that need to be brought to the attention: (i) If a PMU is necessary to be installed on an integrated bus, this might imply that the PMU has to be installed on the ZIB, or on the bus chosen to be integrated with the ZIB, or on both buses. However, it could also mean that the PMU needs to be installed on both buses. Because of this circumstance, a further monitoring test needs to be conducted in order to determine which of the two buses should have the PMU placed on it, and (ii) every time an integration procedure has been carried out, the topology of the system has been modified. When referring to a power grid on a massive scale, this may cause the topology to become more complicated.

OPPP rules without ZIB

Rule 1 A PMU installed at a specific bus has the ability to compute not only the voltage phasors of that bus but also the current phasors of all of the lines that are related to it. In Fig. 2, bus {1} is PMU-equipped bus. Here, V_1, I_{12}, I_{13}, and I_{41} can be unswervingly measured by the employed PMU.

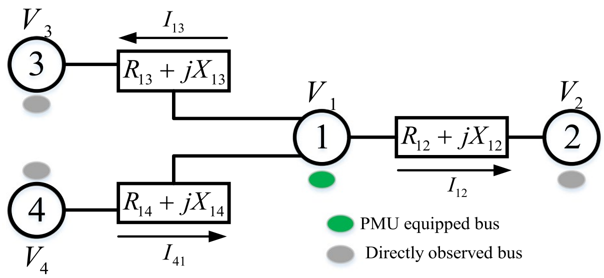

Fig. 2 Modeling PMU placement rule 1

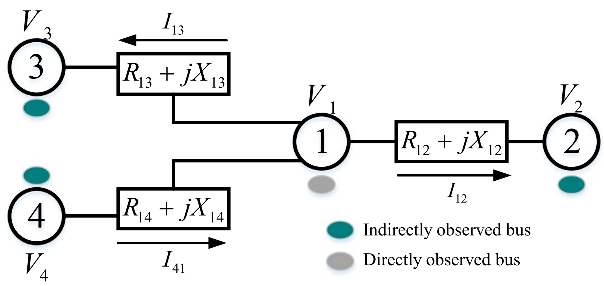

Fig. 3 Modeling PMU placement rule 2

Rule 2 It is possible to calculate the voltage at the other end if the voltage at one end and the line currents of that end are known. Taking into consideration (Fig. 3) and assume that the values of the line current I_{12}, I_{13}, and I_{41} are known, then Ohm's law can be used to compute the voltages at the buses {1}, {3}, and {4}. The values of V_2 and V_3 are the results of V_1 subtracting the potential drop induced by current flowing over the line. Therefore, the values of V_2, V_3, and V_4 are solved as follows:

$$I_{12} = \frac{V_1 - V_2}{R_{12} + jX_{12}} \tag{8}$$

$$V_2 = V_1 - I_{12}(R_{12} + jX_{12}) \tag{9}$$

$$I_{13} = \frac{V_1 - V_3}{R_{13} + jX_{13}} \tag{10}$$

$$V_3 = V_1 - I_{13}(R_{13} + jX_{13}) \tag{11}$$

$$I_{41} = \frac{V_4 - V_1}{R_{14} + jX_{14}} \tag{12}$$

$$V_{41} = V_1 + I_{41}(R_{14} + jX_{14}) \tag{13}$$

Rule 3 If the voltages at both ends of the buses are known, then Ohm's law may be utilized to calculate the line currents that flow between the buses. Given (Fig. 4) that the values of V_1 and V_2 are well known, the line current I_{12} may be determined by using Ohm's law, which is presented in the following form:

$$I_{12} = \frac{V_1 - V_2}{R_{12} + jX_{12}} \tag{14}$$

OPPP rules with ZIB

A bus is known as a ZIB when neither the load nor the generator is connected. As a result, the summation of line currents used at a ZIB is zero. If ZIB, which includes its neighbors, has N_z members, then monitoring $N_z - 1$ buses is enough to turn an unobservable bus into an observable bus. Because of this, while considering ZIB, the number of buses that need to be observed drops by one for each ZIB that is present in the power grid. This, in turn, reduces the minimum number of PMUs that are required for total observability. The following PMU observability criteria are implemented to analyze the topological observability using ZIB:

Rule 4 If there is one bus that is not observable that is adjacent to a ZIB that can be observed, then the bus that cannot be observed can be deemed to be observable. Take, for instance, if the values of V_1, V_2, and V_3 are known, then V_4 may be determined with the use of the KCL at bus {2} which is a ZIB. Refer to Fig. 5, where bus {2} is a ZIB that is observable. Assuming for the moment that the values of V_1, V_2, and V_3 are identified, then the value of line currents I_{12} and I_{23} can be determined by using rule 3 as mentioned above. So, by using KCL at bus {2}, the value of I_{12} is $I_{12} = I_{23} + I_{24}$. For that reason, the value of I_{24} and V_4 can be obtained as follows:

$$I_{24} = \frac{V_2 - V_4}{R_{24} + jX_{24}} \tag{15}$$

$$V_2 - V_4 = I_{24}(R_{24} + jX_{24}) \tag{16}$$

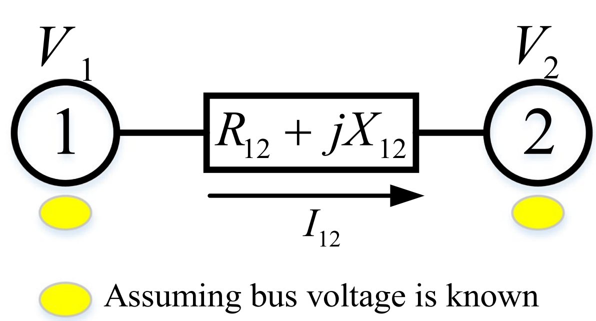

Fig. 4 Modeling PMU placement rule 3

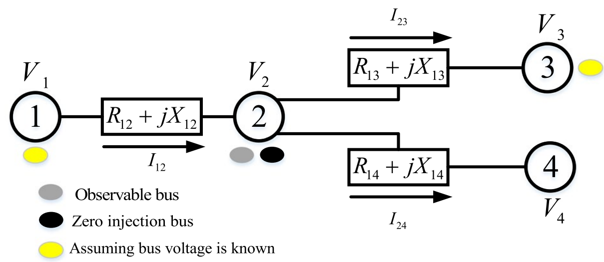

Fig. 5 Modeling PMU placement rule 4

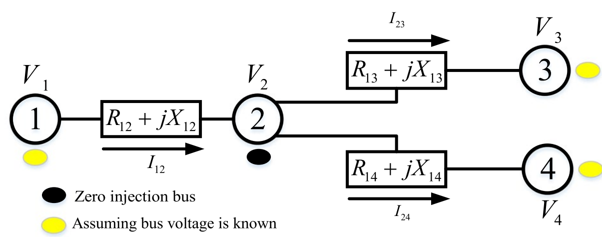

Fig. 6 Modeling PMU placement rule 5

$$V_4 = V_2 - I_{24}(R_{24} + jX_{24}) \tag{17}$$

Rule 5 If the observable buses are linked to ZIB which is unobservable, then the ZIB can be considered as observable. Consider Fig. 6, where bus {2} unobservable ZIB which is connected by all the observable buses such as buses {1}, {3}, and {4}, then the voltage of bus {2} can be obtained as follows:

$$V_2 = V_1 - I_{12}(R_{12} + jX_{12}) \tag{18}$$

$$V_2 = V_3 + I_{23}(R_{23} + jX_{23}) \tag{19}$$

$$V_2 = V_4 + I_{24}(R_{24} + jX_{24}) \tag{20}$$

$$0 = I_{12} - I_{23} - I_{24} \tag{21}$$

Algorithmic perspective of BAPSO in OPPP

Refer to [23], Eqs. (22) and (23) are used to update the velocity of a particle i at each iteration m in the original PSO.

$$\begin{aligned} v_i^{(m+1)} &= w^{(m)} \times v_i^{(m)} + c_1 \times r_1 \times (Pbest_i^{(m)} - y_i^{(m)}) + c_2 \times r_2 \times (Gbest^{(m)} - y_i^{(m)}) \\ y_i^{(m+1)} &= y_i^{(m)} + v_i^{(m+1)} \end{aligned} \tag{22}$$

where $w^{(m)}$ is the inertia weight at the m iteration [24]. The inertia weight in PSO controls the dynamics of flying among particles, and a higher value of this weight leads to global exploration, while a lower value promotes local search. If the inertia weight is set too high, the algorithm may focus too much on exploring new areas and neglect local search, making it challenging to find the exact optimal point. In order to accelerate convergence to the true optimum by balancing global and local exploration, a linearly decreasing inertia weight has been employed:

$$w^{(m)} = (w_{\max} - w_{\min})\left(\frac{M_{\max} - m}{M_{\max}}\right) + w_{\min} \tag{23}$$

The values of inertia weight are of $w_{\max} = 0.9$ and $w_{\min} = 0.4$, and $M_{\max}$ is the maximum number of iterations used in PSO [24, 25]. By introducing a virtual mass to stabilize the motion of the particles, the algorithm is anticipated to have a faster convergence rate. A velocity threshold is introduced [26]:

$$\begin{cases} \mathit{if}\ v_{ij}(m+1) > v_j^{\max} \text{ then } v_{ij}(m+1) = v_j^{\max} \\ \mathit{if}\ v_{ij}(m+1) < -v_j^{\max} \text{ then } v_{ij}(m+1) = -v_j^{\max} \end{cases} \tag{24}$$

where $v_{ij}(m+1)$ is the velocity component of the ith particle along the jth direction at the $(m+1)$th iteration of the algorithm, and $v_j^{\max}$ is the maximum absolute value of velocity allowed along the same jth direction in the parameter space. The adaptation of the inertia weight allows the swarm to achieve convergence with greater accuracy and efficiency as compared to the original PSO. r_1, r_2 are random vectors from the uniform distribution in the range [0, 1] to maintain the swarm diversity. The acceleration constants are $c_1 = c_2 = 2$ called cognitive parameters, so that c_1r_1 and c_2r_2 ensure that the particles would overfly the target about half the time.

The present study presents a meta-heuristic optimization algorithm named BAPSO, which builds upon the BPSO algorithm by incorporating global and local topologies. The BPSO algorithm is known to face the issue of premature convergence and tends to get stuck in local minima. However, the newly introduced mutation strategies in BAPSO can effectively prevent agents from being quickly trapped in local optima, especially when dealing with complex combinatorial problems. BAPSO has the capability to explore the entire solution space for a global search and conduct a local search, leading to the identification of global minima [27]. The evolutionary equation of BAPSO is as follows:

$$\begin{aligned} v_i^{(m+1)} &= a \times (w^{(m)} \times v_i^{(m)} + c_1 \times r_1 \times (Pbest_i^{(m)} - y_i^{(m)}) + c_2 \times r_2 \times (Gbest^{(m)} - y_i^{(m)})) \\ y_i^{(m+1)} &= y_i^{(m)} + v_i^{(m+1)} \end{aligned} \tag{25}$$

where a is the acceleration factor. Compared to the conventional PSO, the evolution equation of BAPSO involves an additional parameter "a," while it includes one more parameter "w" than the PSO with the contraction factor. Despite this, BAPSO has demonstrated impressive results in solving complex OPPP for large-scale inter-power grids within a reasonable time. The proposed BAPSO in this study shares a similar structure with PSOCF. The equation of PSOCF is given as follows [28]:

$$v_i^{(m+1)} = \lambda \times (w^{(m)} \times v_i^{(m)} + c_1 \times r_1 \times (Pbest_i^{(m)} - y_i^{(m)}) + c_2 \times r_2 \times (Gbest^{(m)} - y_i^{(m)}))$$
$$y_i^{(m+1)} = y_i^{(m)} + v_i^{(m+1)} \tag{26}$$

where contraction factor $\lambda = 2k/\left|2 - \phi - \sqrt{\phi(\phi - 4)}\right|$, $k = [0, 1]$, $\phi = c_1 + c_2$. For $c_1 = 3.5$, $c_2 = 0.4$, $\lambda = 2k/\left|2 - \phi - \sqrt{\phi(\phi - 4)}\right| = 2k/\left|2 - 3.9 - \sqrt{3.9(3.9 - 4)}\right|$ does not exist. The PSOCF could not be used, but BAPSO in this work is used for solutions of the OPPP of large-scale inter-power grids and obtained satisfying results. For $c_1 = 2.05$, $c_2 = 2.05$, $\lambda = 2k/\left|2 - \phi - \sqrt{\phi(\phi - 4)}\right| = 2(0 \sim 1)/\left|2 - 4.1 - \sqrt{4.1(4.1 - 4)}\right| = 0 \sim 0.73$. The acceleration factor a is outside the space of the contraction factor λ, and the optimization of BAPSO in this article is performed as usual. Although BAPSO is similar to PSOCF, PSOCF did not have good adaptability as BAPSO [27]. The pseudo-code of the BAPSO is as follows:

Algorithm 1: Pseudo-code for BAPSO

Objective function $f(\overrightarrow{y})$, $\overrightarrow{y} = [y_1, y_2, ..., y_n]^T$
Initialize locations y_i and velocity v_i of n particles
Find *Gbest* from min $\{f(y_1), ..., f(y_n)\}$ at $(m = 0)$
while (criterion)
$m = m + 1$(pseudo time or iteration counter)
For loop over all n particles and all j dimensions
Generate new velocity $v_i^{(m+1)} = a \times (w^{(m)} \times v_i^{(m)} + c_1 \times r_1 \times (Pbest_i^{(m)} - y_i^{(m)}) + c_2 \times r_2 \times (Gbest^{(m)} - y_i^{(m)}))$
Calculate new locations $y_i^{(m+1)} = y_i^{(m)} + v_i^{(m+1)}$
Assess the objective functions at new positions y_i^{m+1}
Determine the present optimum for every particle *Pbest*
end ε
Find the current global best *Gbest*
end while
output the final results *Pbest* and *Gbest*

PSO was originally intended to handle unconstrained optimization, but it has the potential to solve constrained problems with modifications. To locate the global minimum while accounting for constraints, BAPSO employs a constraint-handling approach that updates both a particle's best position and the swarm's global best position. To steer the search toward the feasible area, a feasibility term is included, which determines the extent of the overall constraint violation. The choice of the global best (Gbest) topology in BAPSO depends on the dimension of the search space. In order to enable BAPSO to work with binary problems, the initial Gbest is represented as a binary column vector [25]. The population size is selected according to the network size [29]. The initial inertia parameter could be selected as $w = (0.9 - 0.7 \times rand)$.

The objective is evaluated with a number of moving particles at each iteration. As observed, the BAPSO starts with the iteration to find the global minimum point, whereas the velocity tends to go into $v_{\max}$ or $-v_{\max}$. The value of $v_{\max}$ is carefully selected [25]. When the size is insufficient, the algorithm can get stuck in a local minimum or have to perform more iterations to arrive at the correct solution. The particle is positioned within the binary search space [27], and its current velocity and position impact its future position. The BAPSO is capable of conducting both global and local searches

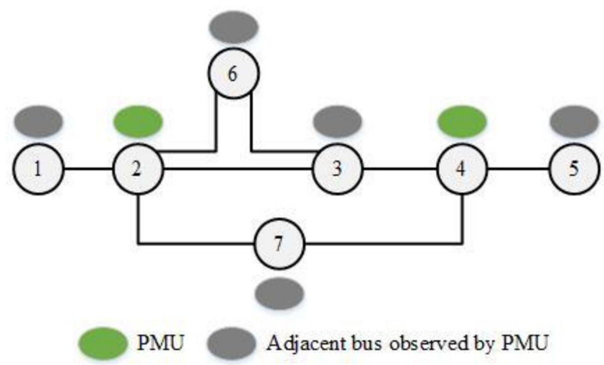

Fig. 7 Seven-bus system

0	1	0	1	0	0	0
1	2	3	4	5	6	7

Fig. 8 Structure of the particle

of the solution space without being confined to local minimum points. To attain better convergence, an inertia weight is utilized to maintain a balance between global and local searches.

Particles

For the OPP problem, every particle has a promising solution. The objective of this work is to determine the optimal minimum number and strategic locations of PMUs to maximize the observability of the power grid. As a result, the configuration of each particle is designed to indicate the availability of PMUs on a particular bus. When determining the OPPP for a 7-bus system (as shown in Fig. 7), the construction of each particle is depicted in Fig. 8, which can be found below. Each dimension of the power grid is linked to a specific bus, and each particle is developed according to these dimensions. A value of {1} at bus {2} indicates that a PMU is installed at that bus, while a value of {0} denotes that there is no PMU installed at bus {2}.

Redundancy measurement

In order to determine the most effective sets of PMU placements, the BOI and SORI redundancy measurement concepts, as described in reference [30], are utilized. BOI refers to the number of times a particular PMU observes a bus, while SORI is the sum of all BOI values. The solution sets that have the least number of PMUs and the greatest sum of BOI, represented by SORI, are considered to be the most optimal. The BOI is the performance metric, which can be calculated using Eq. (27), while Eq. (28) shows how to calculate SORI.

$$BOI = A_{i,k} \cdot N_{PMU}^{T} \tag{27}$$

$$SORI = \sum_{i=1}^{N_{PMU}} A_{i,k}.N_{PMU}^{T} \tag{28}$$

Fitness function

The BAPSO involves particles that carry potential solutions to the OPPP, and in order to determine the best solution, a fitness function is used to evaluate each solution during the investigation. The fitness function must meet three important criteria: ensuring power grid observability, determining the minimum number of PMUs needed for full observability, and measuring redundancy. Following these guidelines, the fitness function for identifying the desired target can be expressed as shown in [22].

$$Fit(Z) = \min\left\{\underbrace{(w_1 \times N_{obs})}_{Observability} + \underbrace{(w_2 \times N_{PMU})}_{Number\ of\ PMUs} + C \times R_1\right\} \tag{29}$$

where w_1 $(= -2)$, w_2 $(= 1)$, and C $(= 0.01)$ are the weight parameters, N_{obs} is the total number of a bus which is observable, N_{PMU} is the number of PMUs equipped bus, and R_1 is the redundancy measurement. The fitness function described in Eq. (29) is comprised of three components: (i) the count of observable buses, (ii) the count of PMUs, and (iii) the redundancy measurement. It is important to highlight that the first component determines the number of buses that can be monitored through the placement of installed PMUs. The value of N_{obs} can be given as follows:

$$N_{obs} = \left|y \in BOI\middle|y \neq 0\right| \tag{30}$$

Additionally, the second component determines the quantity of PMUs, which can be interpreted as follows:

$$N_{\mathrm{PMU}} = Y^{T}Y \tag{31}$$

Moreover, regarding the third component, the value of redundancy measurement is established by:

$$R_1 = (R - BOI)^{T} \times (R - BOI) \tag{32}$$

Results and discussion

The OPPP is solved using a modified particle swarm optimization approach in this study. The traditional BPSO method is limited by premature convergence and is prone to get stuck in local minima. The proposed BAPSO method, on the other hand, can carry out both global and local searches to locate global minima. It effectively prevents agents from quickly becoming trapped in local optima, which is particularly useful in addressing complex combinatorial problems.

In order to implement the proposed method effectively, it is necessary to determine the appropriate parameter values such as population size. To this end, various trial runs have been conducted on all the test systems studied for solving the OPPP, and the optimal results are presented here. The population size is four times the

Table 1 Configuration settings for the optimization method

Parameter names	Values
Population size (pop)	$n \times 4$
Cognitive parameter (c_1)	2.04
Social parameter (c_2)	2.04
Random number (r_1 and r_2)	$0 \leq r_i \leq 1, i = 1, 2$
Inertia parameter ($w_{\max}$ and $w_{\min}$)	0.9–0.4

Table 2 Optimum locations of PMU for 14-bus under normal operations

# of sets	Optimal locations of PMUs	# of PMUs	BOI	SORI	Comp. time (s)
1	2, 6, 7, 9	4	1, 1, 1, 3, 2, 1, 2, 1, 2, 1, 1, 1, 1, 1	19	0.97
2	2, 7, 11, 13	4	1, 1, 1, 2, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1	16	0.81
3	2, 7, 10, 13	4	1, 1, 1, 2, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1	16	1.06

number of buses, which is sufficient for solving the OPPP in the present work. The maximum number of iterations has been set to 250 for smaller systems such as IEEE 14-bus, IEEE 30-bus, New England 39-bus, and IEEE 57-bus systems, while it is 1000 for larger systems such as IEEE 118-bus, IEEE 300-bus, and NRPG 246-bus systems. MATLAB R2013a software was used to conduct the simulations, and the computer used had an Intel Core i3-5005U (2.0 GHz, 3 MB L3 Cache) processor and 8 GB of RAM. The number and location of radial and ZIBs are shown in Appendix "Number and location of a radial and zero injection buses," while the connection of ZIBs is displayed in Appendix "Connection of ZIBs." The parameter values used in the proposed method for the PMU placement problem are listed in Table 1.

The parameter values used to unravel the OPPP were carefully selected through extensive testing to ensure feasible performance. The proposed BAPSO algorithm was found to converge faster than the standard BPSO algorithm for every bus system. The results obtained were satisfactory, and the proposed method achieved adequate computational time, which was only slightly longer than the standard BPSO algorithm. Interestingly, the computational time was found to be superior to that of existing studies. Additionally, the consideration of ZIBs and radial buses from the OPPP minimized the number of PMUs necessary for the entire power network observability.

According to Table 2, it is possible to ensure the observability of the power grid under normal operation for the standard 14-bus system by placing PMUs in the optimum locations and with the minimum number required. After considering the number of trials and with redundancy measurement, it is found that buses {2, 6, 7, and 9} are the best promising set for the OPPP. Here, solution set 2 has a maximum number of SORI, that is, 19. The entries of BOI signify that how many numbers of times the PMU-equipped bus observes each bus, and it is also explained in Sect. "Redundancy measurement."

Table 3 provides the details on the minimum number and optimum locations of PMUs required to achieve full observability of the power grid during normal operations for the standard 30-bus system. Here, {1, 5, 10, 11, 12, 18, 24, 26, 28, and 30} are the best set for PMU location.

Bus number {2, 6, 9, 10, 13, 16, 17, 19, 20, 22, 23, 25, and 29} is the most promising optimal locations of PMU as depicted in Table 4 for NE 39-bus system to make power grid utterly observable during the normal operating condition.

For the 57-bus system, Table 5 displays the crucial locations of PMUs required for complete observability of the power grid during normal operations. These locations include {1, 4, 9, 20, 24, 27, 29, 30, 32, 36, 38, 41, 44, 46, 51, 54, and 57}.

The solution to the OPPP may be found in Table 6, and it is presented here for 118-bus, 246-bus, and 300-bus systems accordingly.

Table 7 displays the OPPP solution for all test systems. Section "Modeling of ZIB" provides a clear explanation of ZIB modeling. To fully observe the 14-, 30-, 39-, 57-, 118-, 246-, and 300-bus networks in this location, respectively, a total of 4, 10, 13, 17, 32, 70, and 87 buses are needed.

Table 7 indicates that the number of PMUs required for achieving maximum observability increases with the expansion of the power grid. As the size of the network increases, computational time also increases. Table 8 displays the optimal position and minimum number of PMUs needed, taking into account ZIBs. It is worth mentioning

Table 3 Optimum locations of PMU for 30-bus under normal operations

# of sets	Optimal locations of PMUs	# of PMUs	BOI	SORI	Comp. time (s)
1	1, 5, 8, 11, 12, 16, 19, 22, 24, 27	10	1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 1, 2, 2, 1, 1, 1, 1, 1	36	3.42
2	1, 7, 8, 11, 12, 16, 19, 22, 24, 27	10	1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 1, 2, 2, 1, 1, 1, 1, 1	36	2.94
3	1, 5, 11, 12, 17, 19, 21, 24, 27, 28	10	1, 2, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 1, 1, 2, 1, 1, 1, 1, 1	35	2.96
4	3, 6, 7, 11, 13, 15, 16, 19, 21, 25, 30	11	1, 1, 1, 2, 1, 2, 2, 1, 2, 2, 1, 3, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1	39	2.92
5	**1, 5, 10, 11, 12, 18, 24, 26, 28, 30**	**10**	**1, 2, 1, 1, 1, 2, 1, 1, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 2, 1, 1, 2, 1, 2, 1, 1, 1**	**37**	**2.43**
6	3, 5, 8, 11, 13, 14, 17, 19, 21, 23, 27	11	1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 2, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1	33	2.45
7	3, 5, 11, 13, 15, 16, 20, 22, 27, 28	10	1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1	33	2.91
8	3, 5, 10, 11, 12, 18, 24, 26, 28, 30	10	1, 1, 1, 2, 1, 2, 1, 1, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 2, 1, 1, 2, 1, 2, 1, 1, 1	37	3.04
9	1, 7, 8, 11, 12, 16, 19, 21, 23, 27	10	1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 1, 1, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1	34	2.82
10	1, 7, 11, 12, 17, 19, 22, 23, 27, 28	10	1, 1, 1, 1, 1, 2, 1, 1, 1, 2, 1, 1, 1, 1, 2, 2, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1	35	2.61

Bold highlighted values are results that are considered the best in terms of time efficiency. They represent optimal outcomes achieved swiftly and efficiently within a given context

Table 4 Optimum locations of PMU for 39-bus under normal operations

# of sets	Optimal locations of PMUs	# of PMUs	BOI	SORI	Comp. time (s)
1	2, 6, 9, 12, 14, 16, 17, 29, 32, 33, 34, 35, 36, 37	14	1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 2, 1, 2, 2, 2, 1, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1	46	7.45
2	2, 3, 6, 9, 12, 14, 17, 20, 22, 23, 25, 32, 38	13	1, 3, 2, 2, 1, 1, 1, 1, 1, 1, 2, 1, 2, 1, 1, 1, 1, 2, 1, 1, 1, 2, 2, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1	49	7.88
3	2, 6, 9, 10, 13, 14, 17, 19, 20, 22, 23, 25, 29	13	1, 2, 1, 1, 1, 1, 1, 1, 1, 2, 2, 1, 3, 2, 1, 2, 1, 1, 2, 2, 1, 2, 2, 1, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1	52	7.93
4	2, 6, 9, 12, 14, 17, 19, 20, 22, 23, 29, 31, 32, 37	14	1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 2, 1, 2, 1, 1, 2, 1, 1, 2, 2, 1, 2, 2, 1, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1	49	7.69
5	2, 6, 9, 10, 12, 14, 17, 19, 20, 22, 23, 25, 29	13	1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 3, 1, 3, 1, 1, 2, 1, 1, 2, 2, 1, 2, 2, 1, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1	51	7.31
6	**2, 6, 9, 10, 13, 16, 17, 19, 20, 22, 23, 25, 29**	**13**	**1, 2, 1, 1, 1, 1, 1, 1, 1, 2, 2, 1, 2, 1, 1, 3, 2, 1, 3, 2, 2, 2, 2, 2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1**	**55**	**7.92**
7	4, 6, 13, 16, 17, 22, 25, 29, 30, 32, 33, 34, 36, 39	14	1, 2, 1, 1, 2, 1, 1, 1, 1, 2, 1, 1, 1, 2, 1, 2, 2, 1, 2, 1, 2, 1, 2, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1	49	8.41
8	2, 5, 6, 9, 13, 16, 17, 19, 20, 22, 23, 29, 32, 37	14	1, 1, 1, 1, 2, 2, 1, 2, 1, 2, 1, 1, 1, 1, 1, 3, 2, 1, 3, 2, 2, 2, 2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1	54	7.14
9	2, 6, 9, 10, 12, 14, 17, 20, 22, 23, 25, 27, 29	13	1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 3, 1, 3, 1, 1, 1, 2, 1, 1, 1, 1, 2, 2, 1, 2, 3, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1	51	7.73
10	2, 6, 9, 14, 17, 19, 22, 23, 25, 29, 32, 34, 37	13	1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 2, 1, 2, 2, 1, 3, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1	48	7.59

Bold highlighted values are results that are considered the best in terms of time efficiency. They represent optimal outcomes achieved swiftly and efficiently within a given context

that the inclusion of ZIBs in simulations reduces the number of PMUs required for observing all buses. For instance, in a 14-bus system under normal operating conditions, four PMUs are required for maximum network observability, but with the consideration of ZIB, only three PMUs are needed.

Tables 9 and 10 compare the results obtained from the BAPSO technique with those obtained from Guo [31], Chakrabarti et al. [32], Milosevic et al. [33], Manousakis et al. [34], and Sodhi et al. [35] for IEEE 14-, 30-, 57-, 118-, NE 39-, NRPG 246-, and 300-bus systems, with and without ZIBs, respectively. In this study, the BAPSO technique was employed to determine the optimal number of PMUs and their positions while maximizing redundancy measurement, ensuring full observability of the power grid. The BAPSO approach was applied to IEEE networks, and the results were compared with those obtained using various programming methods proposed in the previous literature. The comparative analysis demonstrates that the BAPSO technique provides alternative methods where the objective function takes a minimal value in full agreement with the ones defined by the current programming techniques for each case study.

Table 11 presents a comparison between the computational time of the proposed method and the results obtained from using the BPSO algorithm in recent studies.

Table 5 Optimum locations of PMU for 57-bus under normal operations

# of sets	Optimal locations of PMUs	# of PMUs	BOI	SORI	Comp. time (s)
1	1, 4, 9, 20, 23, 27, 29, 30, 32, 36, 39, 41, 44, 46, 47, 50, 54	17	1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 2, 1	66	14.26
2	1, 4, 7, 10, 18, 21, 24, 26, 29, 30, 32, 36, 41, 45, 46, 49, 54, 57	18	1, 1, 1, 2, 1, 2, 2, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 2, 1, 1, 1, 1, 1, 2, 2, 2, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1	67	14.52
3	1, 4, 7, 13, 20, 22, 26, 29, 30, 33, 35, 36, 41, 45, 47, 51, 54, 57	18	1, 1, 1, 1, 1, 2, 2, 1, 1, 1, 2, 1, 1, 1, 3, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1	67	14.55
4	1, 4, 9, 20, 23, 27, 29, 30, 32, 36, 38, 41, 45, 46, 50, 54, 57	17	1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 2, 1	66	13.64
5	**1, 6, 12, 15, 19, 22, 24, 25, 28, 32, 36, 42, 43, 45, 47, 50, 53, 54, 57**	**19**	**2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 3, 2, 2, 1, 1, 1, 1, 1, 2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 2, 2, 1, 2, 1**	**71**	**13.51**
6	1, 4, 9, 14, 20, 23, 27, 29, 30, 32, 36, 39, 42, 43, 44, 48, 50, 54	18	1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 2, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 2, 2, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 1, 1	66	14.29
7	1, 4, 9, 20, 24, 25, 28, 29, 32, 36, 38, 41, 45, 46, 51, 53, 57	17	1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1	63	15.40
8	1, 4, 9, 20, 23, 25, 27, 29, 32, 36, 41, 45, 46, 48, 50, 54, 57	17	1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 2, 1	63	14.64
9	1, 4, 9, 20, 24, 27, 29, 30, 32, 36, 38, 41, 44, 46, 51, 54, 57	17	1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 1	67	15.20
10	1, 4, 9, 20, 24, 27, 29, 30, 32, 36, 38, 41, 44, 46, 51, 54, 57	17	1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 1	67	14.00

Bold highlighted values are results that are considered the best in terms of time efficiency. They represent optimal outcomes achieved swiftly and efficiently within a given context

Table 6 Optimum locations of PMU for 118-bus, 246-bus, and 300-bus under normal operations

Test systems	Optimal locations of PMUs	# of PMUs	BOI	SORI	Comp. time (s)
118-bus	3, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 40, 45, 49, 52, 56, 62, 64, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114	32	1, 1, 3, 1, 2, 1, 1, 2, 1, 1, 2, 2, 1, 2, 2, 2, 2, 1, 2, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 2, 1, 1, 3, 1, 2, 2, 1, 2, 1, 1, 2, 1, 1, 1, 1, 2, 1, 2, 1, 1, 2, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 2, 2, 1, 1, 3, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 3, 2, 1, 2, 3, 1, 1, 3, 1, 3, 1, 1, 1, 1, 1, 2, 1, 1, 3, 2, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1	164	44.72
246-bus	6, 7, 10, 11, 21, 24, 29, 34, 40, 44, 48, 49, 54, 56, 61, 62, 63, 65, 73, 74, 75, 80, 83, 91, 93, 94, 95, 98, 100, 101, 106, 109, 117, 121, 122, 125, 126, 128, 129, 132, 133, 134, 140, 141, 142, 147, 157, 158, 160, 167, 168, 174, 180, 181, 185, 187, 190, 191, 194, 199, 201, 202, 203, 215, 216, 219, 234, 235, 243, 245	70	2, 1, 1, 1, 1, 2, 2, 2, 1, 1, 1, 1, 2, 1, 1, 2, 2, 2, 1, 2, 2, 1, 1, 2, 1, 1, 2, 1, 1, 1, 1, 1, 1, 2, 1, 1, 2, 1, 1, 1, 1, 2, 1, 3, 1, 2, 1, 1, 2, 1, 1, 1, 1, 3, 2, 3, 1, 1, 2, 1, 1, 2, 1, 1, 4, 1, 3, 1, 1, 1, 2, 1, 2, 2, 2, 1, 1, 1, 1, 3, 1, 2, 2, 1, 1, 3, 1, 2, 1, 1, 2, 1, 1, 1, 2, 1, 2, 1, 1, 2, 1, 1, 1, 2, 2, 1, 1, 1, 3, 2, 2, 1, 2, 1, 1, 1, 1, 1, 1, 1, 3, 2, 1, 1, 1, 2, 1, 1, 2, 2, 2, 3, 1, 2, 1, 2, 2, 1, 2, 2, 1, 1, 1, 1, 1, 1, 2, 1, 1, 2, 1, 1, 1, 2, 1, 1, 2, 4, 1, 3, 1, 3, 1, 1, 3, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 3, 1, 2, 1, 2, 1, 1, 1, 3, 1, 1, 1, 1, 1, 1, 2, 1, 2, 1, 2, 2, 2, 1, 1, 1, 1, 1, 1, 2, 1, 3, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 2, 1, 1, 1, 1, 1, 3, 1, 1, 1, 2, 1, 2, 1	351	181.01
300-bus	1, 2, 3, 11, 12, 15, 17, 20, 22, 23, 25, 27, 33, 37, 38, 43, 48, 49, 53, 54, 55, 59, 60, 62, 64, 65, 68, 71, 73, 79, 83, 85, 86, 88, 89, 93, 98, 99, 101, 103, 109, 111, 112, 113, 116, 118, 119, 122, 132, 135, 138, 143, 145, 152, 157, 163, 167, 173, 177, 183, 187, 189, 190, 193, 196, 202, 204, 209, 210, 211, 213, 216, 217, 219, 224, 225, 228, 237, 267, 268, 269, 270, 272, 273, 274, 276, 294	87	2, 2, 3, 1, 1, 1, 2, 2, 1, 2, 1, 2, 1, 1, 2, 1, 2, 2, 1, 3, 1, 2, 3, 2, 1, 1, 1, 1, 2, 1, 2, 2, 1, 1, 2, 1, 2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 2, 2, 3, 1, 1, 1, 2, 2, 2, 2, 1, 2, 2, 3, 2, 2, 1, 3, 2, 2, 2, 3, 1, 1, 2, 1, 3, 1, 1, 1, 1, 1, 2, 1, 2, 2, 4, 2, 3, 1, 1, 2, 2, 2, 1, 1, 2, 1, 2, 1, 1, 2, 2, 1, 1, 2, 1, 1, 1, 1, 2, 2, 2, 1, 2, 2, 2, 1, 1, 4, 1, 1, 2, 1, 2, 1, 2, 2, 1, 1, 2, 1, 2, 1, 1, 1, 1, 2, 1, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 2, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 2, 1, 2, 1, 2, 2, 2, 1, 1, 1, 2, 1, 1, 2, 3, 1, 1, 1, 2, 2, 1, 1, 1, 1, 1, 1, 2, 2, 1, 1, 1, 1, 1, 1, 1, 3, 2, 1, 1, 1, 2, 1, 1, 1, 2, 1, 1, 3, 2, 1, 2, 2, 1, 3, 1, 1, 1, 1, 2, 1, 1, 2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 3, 2, 1, 2, 2, 2, 2, 3, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 2, 1, 1, 1, 1, 1, 1	427	231.82

The study found that an increase in the number of buses resulted in a longer computational time. However, the proposed approach significantly outperformed the previous studies in terms of computational time. This demonstrates that the proposed

Table 7 Optimum locations of PMUs under normal operations

Test systems	# of PMU	Locations of PMU	SORI	Comp. time (s)
14-bus	4	2, 6, 7, 9	19	0.97
30-bus	10	1, 5, 10, 11, 12, 18, 24, 26, 28, 30	37	2.43
39-bus	13	2, 6, 9, 10, 13, 16, 17, 19, 20, 22, 23, 25, 29	55	7.92
57-bus	17	1, 4, 9, 20, 24, 27, 29, 30, 32, 36, 38, 41, 44, 46, 51, 54, 57	67	14.00
118-bus	32	3, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 40, 45, 49, 52, 56, 62, 64, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114	164	44.72
246-bus	70	6, 7, 10, 11, 21, 24, 29, 34, 40, 44, 48, 49, 54, 56, 61, 62, 63, 65, 73, 74, 75, 80, 83, 91, 93, 94, 95, 98, 100, 101, 106, 109, 117, 121, 122, 125, 126, 128, 129, 132, 133, 134, 140, 141, 142, 147, 157, 158, 160, 167, 168, 174, 180, 181, 185, 187, 190, 191, 194, 199, 201, 202, 203, 215, 216, 219, 234, 235, 243, 245	351	181.01
300-bus	87	1, 2, 3, 11, 12, 15, 17, 20, 22, 23, 25, 27, 33, 37, 38, 43, 48, 49, 53, 54, 55, 59, 60, 62, 64, 65, 68, 71, 73, 79, 83, 85, 86, 88, 89, 93, 98, 99, 101, 103, 109, 111, 112, 113, 116, 118, 119, 122, 132, 135, 138, 143, 145, 152, 157, 163, 167, 173, 177, 183, 187, 189, 190, 193, 196, 202, 204, 209, 210, 211, 213, 216, 217, 219, 224, 225, 228, 237, 267, 268, 269, 270, 272, 273, 274, 276, 294	427	231.82

Table 8 Optimum PMUs placement with ZIBs

Test systems	# of PMU	Locations of PMU	SORI	Comp. time (s)
14-bus	3	2, 6, 9	16	0.97
30-bus	7	1, 5, 10, 12, 15, 20, 30	36	2.13
39-bus	8	3, 8, 13, 16, 20, 23, 25, 29	43	6.47
57-bus	11	1, 6, 13, 19, 25, 29, 32, 38, 51, 54, 56	60	12.59
118-bus	28	3, 8, 11, 12, 17, 21, 27, 31, 32, 34, 37, 40, 45, 49, 52, 56, 62, 72, 75, 77, 80, 85, 86, 90, 94, 102, 105, 110	156	41.69
246-bus	62	3, 6, 7, 10, 11, 18, 21, 22, 24, 29, 34, 40, 44, 48, 49, 50, 57, 65, 78, 80, 83, 85, 91, 92, 93, 96, 98, 101, 106, 109, 113, 117, 121, 125, 128, 132, 134, 140, 141, 142, 153, 157, 158, 160, 163, 168, 173, 181, 185, 187, 190, 191, 194, 199, 201, 202, 203, 219, 229, 235, 242, 245	323	160.07
300-bus	69	1, 2, 3, 11, 15, 17, 20, 23, 24, 26, 37, 41, 43, 44, 55, 57, 61, 63, 70, 71, 72, 77, 97, 104, 105, 108, 109, 114, 119, 120, 122, 126, 139, 140, 145, 152, 154, 155, 166, 175, 178, 184, 187, 188, 198, 205, 210, 211, 214, 216, 223, 225, 229, 231, 232, 234, 237, 238, 240, 245, 249, 267, 268, 269, 270, 272, 274, 276, 294	393	289.55

Table 9 Comparison of obtained results with existing methods without a ZIB

Test system	Guo	Chakrabarti et al.	Milosevic et al.	Manousakis et al.	Sodhi et al.	Proposed method
14-bus	4	4	4	4	4	4
30-bus	10	10	10	10	–	10
39-bus	–	13	13	–	15	13
57-bus	17	–	–	17	–	17
118-bus	32	32	–	–	–	32
246-bus	–	–	–	–	70	70
300-bus	–	–	–	–	–	87

"–" means not reported

Table 10 Comparison of obtained results with existing methods with ZIB

Test system	Guo	Chakrabarti et al.	Milosevic et al.	Manousakis et al.	Sodhi et al.	Proposed method
14-bus	3	3	3	3	3	3
30-bus	7	7	7	7	–	7
39-bus	–	8	8	–	8	8
57-bus	11	–	–	11	–	11
118-bus	28	29	–	–	–	28
246-bus	–	–	–	–	62	62
300-bus	–	–	–	–	–	69

"–" means not reported

Table 11 Comparison of computational time with existing methods ignoring and considering ZIBs

Test system	Modified BPSO [12]		Improved PSO [14]		Proposed methods	
	Ig. ZIBs	Cons. ZIBs	Ig. ZIBs	Cons. ZIBs	Ig. ZIBs	Cons. ZIBs
	Comp. time (s)	Comp. time (s)	Comp. time (s)	Comp. time (s)	Comp. time (s)	Comp. time (s)
14-bus	1.5	1.60	–	–	0.97	0.97
30-bus	6.8	3.60	8.20	12.20	2.92	2.13
39-bus	–	9.00	11.73	27.78	7.92	6.47
57-bus	–	25.80	35.00	33.64	14.00	12.59
118-bus	–	51.00	49.80	96.27	44.72	41.69
246-bus	–	–	–	–	181.01	160.07
300-bus	–	–	–	–	231.82	289.55

"–" means not reported

Ig. and *Cons.* ignoring and considering; *Comp.* computational

approach not only yields high-quality solutions but also operates at a faster computational pace.

Conclusions and scopes of future work

The purpose of this paper is to introduce a novel BAPSO algorithm that incorporates global and local topologies to solve OPPP, to enhance the learning and convergence procedure of classifiers. The proposed algorithms have numerous benefits, such as simplicity, ease of implementation, and the lack of need for algorithm-specific parameters. Instead, they require only common controlling parameters, such as the number of generations, population size, and tuning of the acceleration coefficient. The efficacy of the BAPSO algorithm in achieving an OPPP solution is demonstrated using IEEE bus systems. In binary PSO, the population size is a crucial factor in achieving optimal execution time and solution consistency. However, increasing the population size also increases the total execution time. The study finds that the algorithm's average execution time and performance are directly proportional to the size of the population and the maximum number of iterations. In a large-scale network, conventional BPSO can generate a set of optimum solutions, but it is not feasible within a reasonable timeframe. Conversely, the proposed BAPSO approach offers a fast OPPP solution for large-scale power grids. The results indicate that the proposed algorithms outperform other meta-heuristic algorithms available in the state-of-the-art literature.

The suggested method may be developed further in a number of different ways that may be researched. These include, among others things:

(a) Performance Assessment: Compare the proposed BAPSO technique to other optimization algorithms utilized for PMU allocation. To illustrate the usefulness and efficiency of the suggested technique, this assessment should encompass a wide range of test systems with different sizes and complexity.
(b) Resilience Analysis: Evaluate the suggested allocation method's resilience by taking into account power system parameter uncertainties including demand fluctuations, line outages, and generator failures. Look at the PMU allocation scheme's capacity to adjust to such dynamic events and provide dependable observability under challenging circumstances.
(c) Network Topology Incorporation: Look into incorporating network topology limitations into the PMU allocation procedure. In order to obtain optimal PMU placement, take into account the effects of network structure, such as the presence of radial or meshed networks, and design a strategy that integrates topological considerations.
(d) Investigate the best location for PMUs when using them for wide-area monitoring applications, taking into account local or global power grids. Create a framework that considers geographic and connectivity factors in order to improve situational awareness and system stability in massive power systems.
(e) Cybersecurity Considerations: Examine the possible hazards and vulnerabilities related to the installation of PMUs in the electrical grid. To lessen the danger of cyberattacks and unauthorized access to vital power system infrastructure, look into ways to protect the security and integrity of PMU data and suggest solutions for safe PMU installation.
(f) Cost-Effectiveness Study: Conduct a thorough cost-effectiveness study to assess the financial advantages of the suggested PMU allocation strategy. Think about things such as the price of PMUs, installation, communication setup, and upkeep. Create optimization models that strive to achieve the required level of observability while minimizing the total cost.
(g) Real-Time Implementation: Examine the viability and practicality of putting the suggested PMU allocation technique into use during the real-time operation of the power system. Think about the computational effectiveness, the communication needs, and the SCADA system integration. Create methods for real-time PMU allocation changes and ongoing power system monitoring.
(h) Application to Renewable Energy Integration: The suggested PMU allocation methodology should be expanded to accommodate the unique difficulties involved in integrating renewable energy sources into the power grid, such as solar and wind. Develop methods for the best PMU deployment in grids with a high concentration of renewable energy sources by looking at the effects of distributed generation and intermittent power generation on observability needs.

(i) These suggestions can act as a springboard for more study, enabling the development and improvement of the suggested PMU allocation strategy and eventually advancing the observability and stability of the power grid.

Appendix

Number and location of a radial and zero injection buses

The positions and numbers of radial and zero injection buses for the IEEE 14-, 30-, 57-, 118-, 300-bus, New England 39-bus, and NRPG 246-bus systems are displayed in Table 12.

Table 12 Radial and zero injection buses

Test systems	# of radial buses	# of zero injection buses	Position of radial buses	Position of ZIBs
14-bus	1	1	8	7
30-bus	3	6	11, 13, 26	6, 9, 22, 25, 27, 28
39-bus	9	12	30, 31, 32, 33, 34, 35, 36, 37, 38	1, 2, 5, 6, 9, 10, 11, 13, 14, 17, 19, 22
57-bus	–	15	–	4, 7, 11, 21, 22, 24, 26, 34, 36, 37, 39, 40, 45, 46, 48
118-bus	7	10	10, 73, 87, 111, 112, 116, 117	5, 9, 30, 37, 38, 63, 64, 68, 71, 81
246-bus	34	60	2, 4, 5, 12, 30, 31, 38, 41, 47, 51, 52, 53, 58, 76, 77, 112, 120, 123, 124, 135, 149, 153, 156, 159, 224, 246	51, 53, 54, 56, 58, 61, 62, 63, 69, 70, 71, 72, 73, 74, 75, 80, 81, 86, 102, 103, 104, 107, 122, 126, 129, 131, 147, 154, 155, 167, 175, 179, 180, 183, 209, 210, 211, 212, 213, 214, 215, 216, 217, 221, 222, 226, 229, 230, 231, 232, 233, 234, 236, 237, 238, 239, 240, 241, 243, 244
300-bus	69	65	69, 150, 164, 192, 201, 206, 209, 212, 215, 218, 220, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 275, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 293, 295, 296, 297, 298, 299, 300	4, 7, 12, 16, 18, 23, 28, 29, 30, 33, 36, 39, 40, 52, 54, 56, 57, 62, 65, 70, 71, 72, 73, 82, 94, 95, 96, 107, 108, 109, 110, 111, 112, 113, 123, 129, 130, 137, 139, 143, 144, 145, 147, 148, 153, 172, 173, 174, 189, 191, 198, 205, 216, 219, 223, 245, 246, 266, 270, 271, 272, 273, 276, 291

"–" entry means not reported

Connection of ZIBs

The connection of ZIB with other buses for all test systems is shown in Table 13.

Table 13 Buses connected with ZIB for all test systems

Test systems	ZIB	Buses connected with ZIB
14-bus	7	4, 8, 9
30-bus	6	2, 4, 7, 8, 9, 10, 28
	9	6, 11, 10
	22	10, 21, 24
	25	24, 26, 27
	27	25, 26, 29
	28	8, 6
39-bus	1	2, 39
	2	1, 3, 25, 30
	5	4, 6, 8
	6	5, 7, 11, 31
	9	8, 39
	10	11, 13, 32
	11	6, 10, 12
	13	10, 12, 14
	14	4, 13, 15
	17	16, 18, 27
	19	16, 20, 33
	22	21, 23, 35
57-bus	4	3, 5, 6, 18
	7	6, 8, 29
	11	9, 13, 41, 43
	21	20, 22
	22	21, 23, 38
	24	23, 25, 26
	26	24, 27
	34	32, 35
	36	35, 37, 40
	37	36, 38, 39
	39	37, 57
	40	36, 56
	45	15, 44
	46	14, 47
	48	47, 49, 38
118-bus	5	4, 3, 6, 8, 11
	9	8, 10
	30	17, 8, 26, 38
	37	35, 33, 34, 38, 39, 40
	38	37, 30, 65
	63	59, 64
	64	63, 61, 65
	68	65, 69, 81, 116
	71	70, 72, 73
	81	68, 80
246-bus	51	54
	53	61
	54	62, 49, 51, 52, 55
	56	71, 44, 45, 46, 55, 25, 80
	58	63

Table 13 (continued)

Test systems	ZIB	Buses connected with ZIB
	61	53, 154
	62	54, 4, 71
	63	58, 70
	69	115, 10, 70, 154
	70	232, 63, 69, 72, 154, 238
	71	56, 3, 62, 72
	72	84, 84, 84, 70, 71, 73
	73	74, 3, 72
	74	73, 86, 88, 104, 246
	75	76, 91
	80	44, 46, 56, 82, 12, 86
	81	97, 101
	86	74, 11, 80
	102	64, 83
	103	83, 85
	104	74, 83
	107	105, 106
	122	112, 121
	126	113, 120, 129
	129	113, 118, 126, 127, 232
	131	109, 10
	147	134, 150, 153
	154	140, 61, 69, 70, 155, 240
	155	141, 154
	167	163, 165, 166
	175	157, 166, 177
	179	160, 238
	180	157, 23
	183	133, 182, 194, 34
	209	185, 212, 213
	210	187, 221
	211	190, 212, 217, 221, 40
	212	191, 209, 211, 213
	213	194, 209, 212, 214, 215, 235
	214	199, 213, 221
	215	27, 213, 33, 221, 239
	216	204, 59, 217, 218
	217	205, 211, 33, 216
	221	222, 210, 211, 214, 215, 32
	222	221, 190, 27
	226	1, 201
	229	233, 116, 17, 119, 121, 136, 35
	230	238, 181, 160
	231	239, 201, 27, 39
	232	70, 118, 129
	233	229, 234, 235, 238, 239
	234	233, 237, 238, 239
	236	97, 235
	237	36, 234

Table 13 (continued)

Test systems	ZIB	Buses connected with ZIB
	238	230, 70, 179, 233, 234
	239	231, 215, 233, 234, 40, 241
	240	139, 154, 235
	241	40, 239
	243	42, 242, 245
	244	42, 245
300-bus	4	16, 3
	7	3, 12, 110, 5, 6
	12	7, 20, 10, 251
	16	4, 36, 15
	18	3, 20, 72
	23	20, 231, 22, 254
	28	27, 36
	29	60, 63, 64, 30
	30	73, 29
	33	36, 255
	36	16, 28, 33, 35, 40
	39	52, 62, 38, 40
	40	36, 68, 39
	52	39, 54
	54	52, 56, 123, 53, 261
	56	54, 55
	57	190, 66, 180
	62	39, 73, 240, 61
	65	64, 66, 69
	68	40, 173, 174, 73
	70	71, 81
	71	70, 72, 234, 83
	72	18, 71, 78
	73	30, 62, 68
	82	79, 80, 83
	94	101, 100
	95	99, 103
	96	97, 138
	107	106, 109, 112
	108	105, 109, 112
	109	107, 108, 111, 130, 146, 147, 110, 129, 263
	110	7, 109
	111	105, 109, 149
	112	107, 108, 116, 147, 148, 150
	113	106, 114, 163
	123	54, 122
	129	3, 109
	130	109, 149

Table 13 (continued)

Test systems	ZIB	Buses connected with ZIB
	137	105, 138, 139
	139	137, 103
	143	141, 142, 134
	144	141, 145
	145	144, 265
	147	106, 109, 112
	148	105, 112, 146
	153	151, 152, 120
	172	184, 187, 175
	173	68, 198, 242
	174	68, 198, 191
	189	175, 177, 178, 179, 168
	191	194, 174, 190
	198	173, 174, 196, 216, 197
	205	203, 210
	216	198, 201, 210, 213, 220
	219	169, 230
	223	222, 225
	245	97, 99
	246	183, 175
	266	31, 270, 271, 273
	270	266, 292, 293, 294, 295, 296
	271	266, 272, 268
	272	271, 297, 298, 268
	273	266, 267, 299
	276	274, 278, 279
	291	268, 269

Abbreviations

BAPSO	Binary accelerated particle swarm optimization
BBA	Branch-and-bound algorithm
BOI	Bus observability index
BPSO	Binary particle swarm optimization
EPSO	Enhanced particle swarm optimization
GA	Genetic algorithm
ILP	Integer linear programming
MILP	Mixed integer linear programming
MIP	Mixed integer programming
OPPP	Optimal PMU placement problem
PMU	Phasor measurement unit
PSO	Particle swarm optimization
SORI	System observability redundancy index
ZIBs	Zero injection buses

List of symbols

c_1 and c_2	Acceleration coefficient
a	Acceleration factor
Y	Binary design variable vector having components y_i which decide the feasibility of PMUs on ith bus
$A_{\text{PMU}} = [A_{i,k}]_{n\times n}$	Binary connectivity matrix that explains the bus-to-bus connection
λ	Contraction factor

$Gbest^{(m)}$	Global best position in the swarm at iteration m
w	Inertia weight
$v_j^{\max}$	Maximum absolute value of velocity allowed along the same jth direction in the parameter space
$M_{\max}$	Maximum number of iterations used in PSO
$w_{\max}$	Maximum value of inertia weight
$V_{\max}$	Maximum velocity
$w_{\min}$	Minimum value of inertia weight
$V_{\min}$	Minimum velocity
n	Number of buses
m	Number of iteration
N_{PMU}	Number of PMUs equipped bus
$Pbest_i^{(m)}$	Personal best position for particle i discovered so far
pop	Population size
r_1 and r_2	Random numbers that are uniformly distributed between [0, 1] to maintain the swarm diversity
R_1	Redundancy measurement
k	Receiving end node
i	Sending end node
H and B_{PMU}	Transformation matrix that may be modified according to the contingency cases W
N_{obs}	Total number of a bus which is observable
c_i	Vector of PMU price coefficients
$v_{ij}(m+1)$	Velocity component of the ith particle along the jth direction at the $(m+1)$th iteration of the algorithm
$v^{(m)}$	Velocity of particle i at iteration m
w_1	Weight parameter for the number of bus observed
w_2	Weight parameter for the number of PMUs
C	Weight parameter value for the measurement redundancy

Acknowledgements
Not applicable.

Author contributions
The paper was designed and developed by RB, with SR approving the article's content and SM providing assistance in revising and rewriting it. All authors have read and approved the final manuscript.

Funding
No funding.

Availability of data and materials
Not applicable.

Declarations

Ethics approval and consent to participate
Not applicable.

Consent for publication
Not applicable.

Competing interests
The authors declare that they have no competing interests.

Received: 2 May 2023 Accepted: 15 August 2023
Published online: 04 September 2023

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