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The robust H_∞ control of stochastic neutral state delay systems

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Abstract

In this paper, the resilient stochastic neutral state delay system with the Markov chain problem is examined. It offers output feedback control based on discrete state space delay and uncertainty that appear under traditional delay-independent conditions and exist under neutral stochastic state delays. Lyapunov theory can be used to solve the formulation $x_c(t) = x(t) - Cx(t-\tau)$ for the intended H_∞ control of a stochastic model. Its stochastically mean square stability is shown using linear matrix inequalities (LMIs). The efficiency of the suggested strategy is shown by simulation results.

Keywords: H_∞ control, Lyapunov function, Neutral type stochastic delay

Introduction

Over the past few decades, neutral type stochastic state delay approach has become one of the most popular control strategies for a class of deterministic strict-feedback nonlinear systems. The neutral type schemes [2–6, 13, 16, 19] are all significant examples. Since stochastic neutral models are essential for many applications in engineering and research, numerous control mechanisms have been developed to ensure probability stability. In this study, robust stochastic state stability was established, and a neutral-type delay system was used in the design process together with H_∞ control.

For stochastic neutral delays, there are two different stability analyses: one is delay-dependent and contains information on delay sizes [3] through [9, 16, 17]; the other is delay-independent and can be applied to delays of any size [13]. Both employ LMI descriptions as part of their optimization techniques. In [10, 11], it is discussed how mixed nonlinear odd-even effects and arbitrary switching effects can be used to produce stochastic stability of finite duration. Time delays and parameter uncertainty are issues with the stochastic model's H_∞ control; these issues were fixed when state feedback controllers could be constructed with the assumption that all state variables were present. The results cannot be applied if portions of the real states are unavailable. In [18] Introduced decomposition matrix technique with Jensen's integral inequality, Peng-Park's integral inequality, Leibniz-Newton formula and proved exponential stability of H_∞ performance level. In this paper, all the inequalities are exist in $x(t) = f(t, x_t)$, do not given any importance for neutral delay. In [22] investigated finite-time bounded (FTB) tracking control for a class of neutral systems. Firstly, the dynamic equation of the tracking error signal is given based on the original neutral system. Then, it combines with the equations of the state vector to construct an error system, where the reference signal and the disturbance signal are fused in a new vector. The error system, input–output finite-time of the closed-loop system is proved stability by utilizing the Lyapunov–Krasovskii functional. The finite-time stability conditions are formulated in linear matrix inequalities (LMIs). It does not involved any neutral delay system.

The conventional delay-independent condition, which calls for a controller design with a uniformly sized time delay, is the only one that is used by the techniques. All physical systems cannot be applied because uniform time delay is not always present. To achieve less conservative delay-dependent conditions, many different approaches have been tried. An efficient stochastic differential system is determining the size of the delay under conservative assumptions of neutral type. The stability issue, however, clearly hinges on time delay because it is well known that a certain delay-independent criterion fails. It is required to show the time delay in the control design in order to produce delay-dependent conditions, which is necessary to get over this block [5, 7, 8, 12, 14, 15, 17–19, 21].

However, Chen et al. [2], Niculescu [19] studied the delay-dependent exponential stability of stochastic systems can similarly to $x(t)$, which is stochastically stable in a deterministic manner. However, the presence of neutral stochastic case makes things more complicated and prevents the direct application of the techniques [6, 9, 17] of all deterministic state variables to a stochastic neutral system. Regardless of the parameter uncertainties, which have not yet been fully investigated, the stability criteria are dependent on dynamic output feedback controllers recommended by disturbance attenuation level; however, it is still an open and difficult problem. In this study, we demonstrated that the cap is stochastic by demonstrating that $x_c(t) = x(t) - Cx(t - \tau), y(t) = f(t, x_t)$.

The paper is organized as follows. In "Preliminary modeling" section contains preliminary results. Some sufficient conditions are constructed for stochastic neutral system which is given in "Main models" section. In "The robust H_∞ control" section, the stability of state delay with H_∞ control of stochastic neutral system is given. In "Numerical examples" section presents two numerical examples to show the validity of the proposed results and some conclusions are drawn in "Conclusions" section.

Notations In this paper, R^n and $R^{n\times m}$ denote, the n dimensional Euclidean space, respectively, and the set of all $n \times m$ real matrices; $L^2[0, \infty)$ is the space of square integrable on the interval $[0, \infty)$. The Euclidean norm $|\cdot|$ is in R^n and $C([-l, 0] : R^n)$ which denote the family of continuous function ϕ form $[-l, 0]$ to R^n with the norm $\|\phi\| = \sup_{-l\leq\theta\leq 0} |\phi(\theta)|$, and I denote the compatible dimension identity matrix. The notation $X \geq Y$ (respectively, $X > Y$) where X and Y are symmetric matrices, which is meant that $X - Y$ is positive semi-definite (respectively, positive definite). The matrix A denotes the transpose of $\lambda_{\max}$ and $\lambda_{\min}$, it stands for eigenvalue of maximum and eigenvalue of minimum respectively and the operator norm is denoted by $\|A\| = \sup\{|Ax| : |x| = 1\} = \sqrt{\lambda_{\max}(A^TA)}$. The notation $(\Omega, \mathcal{F}, \{\mathcal{F}t\}_{t\geq 0}, \mathcal{P})$ is complete probability space with a filtration $\{\mathcal{F}\}_t$ and satisfying the usual conditions (i.e., the filtration contains all P-null set is right continuous). The space $L^P_{\mathcal{F}_t}([-l, 0] : R^n)$ denotes the family of all $\mathcal{F}_r$-measurable $C([-l, 0] : R^n)$-valued random variables. The value $\varphi = \{\varphi(\theta) : -l \le \theta \le 0\}$ is related to $\sup_{-l \le \theta \le 0} E|\varphi(\theta)|^p < \infty$.

Preliminary modeling

Let $\{r(t), t \ge 0\}$ be a right continuous Markov chain on the probability space taking values in a finite state space $S = \{1, 2, \ldots, N\}$, then following transition probabilities are holds [20];

$$P\{r(t+\Delta)=j/r(t)=i\}=\begin{cases}\Delta\Lambda_{ij} + o(\Delta), if\ i \ne j,\\ 1 + \Delta\Lambda_{ij} + o(\Delta), if\ i = j,\end{cases} \tag{1}$$

where $\Delta > 0$ and $\Lambda_{ij} \ge 0$ is the transition rate from i to j if $i \ne j$ while $\Lambda = -\sum_{j \ne i} \Lambda_{ij}$.

If $A^m = [a_{ij}^m]_{n \times n}$ and $A^M = [a_{ij}^M]_{n \times n}$, satisfying $a_{ij}^m \le a_{ij}^M, \forall 1 \le i, j \le n$, the interval matrix $[A^m, A^M]$ is defined by $[A^m, A^M] = \{A = [a_{ij} : a_{ij}^m \le a_{ij} \le a_{ij}^M, 1 \le i, j \le n\}$. If $A, \overline{A} \in R^{n \times n}$, $\|\Delta A\| \le \overline{A}$ is a nonnegative matrix, then $[A \pm \overline{A}]$ denotes the interval matrix $[A - \overline{A}, A + \overline{A}]$. In fact, any interval matrix $[A^m, A^M]$ has a unique representation of the form $[A \pm \overline{A}]$, where $A = (\frac{1}{2})(A^m + A^M)$, then $\overline{A} = (\frac{1}{2})(A^m - A^M)$.

For each $i \in S$ and the interval matrices $[A_i \pm \overline{A_i}], [B_i \pm \overline{B_i}], [C_i \pm \overline{C_i}], [D_i \pm \overline{D_i}]$ are exists in S, then consider n-dimensional stochastic neutral state delay system with Markovian switching structure as

$$\begin{aligned}
&d[x(t) - (A_q(r(t)) + \Delta A_q(r(t)))(t-q)]\\
&\quad = [(A(r(t)) + \Delta A(r(t)))x(t) + (A_h(r(t))\\
&\qquad + \Delta A_h(r(t)))x(t-h) + (B_1(r(t))\\
&\qquad + \Delta B_1(r(t)))v(t)\\
&\qquad + (B_2(r(t)) + \Delta B_2(r(t)))w(t)]\mathrm{dt} + [(\mathrm{E(r(t))}\\
&\qquad + \Delta E(r(t)))x(t) + (E_h(r(t)) + \Delta E_h(r(t)))\\
&\qquad \times x(t-h) + (H_1(r(t)) + \Delta H_1(r(t)))w(t)]\mathrm{d}\omega(\mathrm{t}),\\
&\quad z(t) = (C(r(t)) + \Delta C(r(t)))x(t) + (C_h(r(t))\\
&\qquad + \Delta C_h(r(t)))x(t-h) + (D_1(r(t))\\
&\qquad + \Delta D_1(r(t)))v(t) + (D_2(r(t))\\
&\qquad + \Delta D_2(r(t)))w(t),\\
&\quad x_{t_0} = x(t_0 + \theta) = \phi(\theta), \theta \in [-l, 0],
\end{aligned} \tag{2}$$

where $x(t) \in R^n$ is the state vector, $v(t) \in R^{n \times m}$ is the control input and $w(t) \in R^q$ is an exogenous disturbance, $z(t) \in R^p$ is an output controller, $A, A_h, A_q, B_1, B_2, C, C_h, D_1, D_2$ are known real constant matrices with appropriate dimensions, $q > 0$, $h > 0$ are constant time delays and q may not be equal to h, here $l = \max\{h, q\}$ and $\phi(\theta)$ are continuously differentiable on the interval $[-l, 0]$, $\mathrm{d}\omega(\mathrm{t})$ is a Brownian motion and E, E_h, H_1 are stochastic constant matrices.

Main models

Consider state feedback controller is

$$v(t) = K_i x(t),\ \forall\ K_i \in R^{n \times m},\ i = r(t), \tag{3}$$

to prove stability of system (2), the following assumptions are need. In this case the notation, $A(r(t))$ will be denoted as A_r and so on and time varying uncertainties are assumed as

$$\begin{aligned}
\widetilde{A}_i &= A_r + \Delta A_r, \widetilde{B}_{1i} = B_{1r} + \Delta B_{1r},\\
\widetilde{E}_i &= E_r + \Delta E_r, \widetilde{E}_{h_i} = E_{hr} + \Delta E_{hr},\\
\widetilde{B}_{1i} &= B_{1r} + \Delta B_{1r}, \widetilde{B}_{2i} = B_{2r} + \Delta B_{2r},\\
\widetilde{C}_i &= C_r + \Delta C_r, \widetilde{A}_{q_i} = A_{qr} + \Delta A_{qr},\\
\widetilde{C}_{h_i} &= C_h + \Delta C_h, \widetilde{D}_{1i} = D_{1r} + \Delta D_{1r},\\
\widetilde{D}_{2i} &= D_{2r} + \Delta D_{2r}, \widetilde{H}_{1i} = H_{1r} + \Delta H_{1r},\\
\widetilde{H}_{2i} &= H_{2r} + \Delta H_{2r},
\end{aligned}$$

where $\Delta A_r, \Delta A_{qr}, \Delta B_{1r}, \Delta B_{2r}, \Delta E_{hr}, \Delta C_r, \Delta H_{1r}, \Delta H_{2r}, \Delta D_{1r}, \Delta D_{2r}$ are time varying uncertainty parameters. The resulting of the uncertain closed-loop system (2)–(3) can be written as

$$\begin{aligned}
&d[x(t) - \widetilde{A}_{q_i}(t-q)]\\
&\quad = [\widetilde{A}_i x(t) + \widetilde{A}_{h_i} x(t-h) + \widetilde{B}_{1i} v(t) + \widetilde{B}_{2i} w(t)]\mathrm{dt}\\
&\qquad + [\widetilde{E}_i x(t) + \widetilde{E}_{h_i} x(t-h) + \widetilde{H}_{1i} w(t)]\mathrm{d}\omega(\mathrm{t}),\\
&\quad z(t) = \widetilde{C}_i x(t) + \widetilde{C}_{h_i} x(t-h) + \widetilde{D}_{1i} v(t) + \widetilde{D}_{2i} w(t),\\
&\quad x_{t_0} = x(t_0 + \theta) = \phi(\theta), \theta \in [-l, 0].
\end{aligned} \tag{4}$$

In this case, the controller access are depending on state response $x(t)$ and the jumping process $r(t)$. The solution of system (1)–(4) at time t is $x(t, \phi, r(t), v, w)$ which is related to the initial conditions ϕ and jumping process $r(t)$, and the control input $v(t)$, disturbance $w(t)$, respectively, and $x(0, x_0, r(0), v, w)$ represents solution of system (4) at time $t = 0$.

Definition 3.1 The state stochastic delay system (4) with $q, h \in L^2([-l, 0] : R^n)$, and for every $\phi \in R^n$ exists, then solution $x(t, \phi, r(t), v, w)$ is said to be mean-square stochastically stabilizable if

$$\begin{aligned}
&\lim_{t\to\infty} E\Big\{ \int_0^T (x(t, \phi, r(t), v) - A_{q_i} x(t-q))^T\\
&\qquad \times (x(t, \phi, r(t), v) - A_{q_i} x(t-q))\mathrm{dt} \Big\} \le \mathrm{x}_0^{\mathrm{T}} \overline{\mathrm{P}} \mathrm{x}_0,
\end{aligned}$$

and is said to be robustically stochastic stable with disturbance attenuation γ, if it is mean-square stochastically stabilizable under zero initial conditions of $E\|z\|_{L^2} \le \gamma \|w(t)\|_{L^2}$, then for all nonzero $w \in L^2[0, \infty)$ such that

$$\int_0^\infty E[z(t)^T z(t)]\mathrm{dt} \le \gamma^2 \int_0^\infty \mathrm{w(t)^T w(t)dt}.$$

Let

$$\|w\|_2 = \Big\{ \int_0^{\infty} w^T(t)w(t)\mathrm{dt} \Big\}$$

and

$$\|z\|_2 = \Big\{ \int_0^{\infty} z^T(t)z(t)\mathrm{dt} \Big\},$$

and let G_{zw} denote the system exogenous input $w(t)$ and control output $z(t)$, then H_∞ -norm of the closed-loop system (1)–(4) is equivalent to $\|G_{zw}\| < \gamma$.

The following lemma provides some sufficient condition for stochastic stability of H_∞ control system.

Lemma 3.1 [20] *Let Q_1 be a nonnegative definite symmetric matrix, there exists an interval matrix $A_r, \Delta A_r \in [A \pm \overline{A}]$ such that*

$$\begin{aligned}(A_r + \Delta A_r)Q_1(A_r + \Delta A_r)^T \\ \leq (1+\epsilon)A_r^T Q_1 A_r + (1+\frac{1}{\epsilon})\Delta A_r^T Q_1 \Delta A_r\end{aligned} \tag{5}$$

and

$$\begin{aligned}&(A_r + \Delta A_r + E_r + \Delta E_r)Q_1(A_r + \Delta A_r + E_r + \Delta E_r)^T \\ &\quad \leq A_r^T Q_1 A_r + \Delta A_r^T Q_1 \Delta A_r + E_r Q_1 E_r^T \\ &\qquad + \Delta E_r Q_1 \Delta E_r^T + 3(A_r^T Q_1 A_r + \frac{1}{\epsilon}\Delta A_r^T Q_1 \Delta A_r) \\ &\qquad + 2\epsilon E_r Q_1 E_r^T + \frac{1}{\epsilon} E_r Q_1 E_r^T + \epsilon \Delta A_r^T Q_1 \Delta A_r \\ &\qquad + \frac{2}{\epsilon}\Delta E_r^T Q_1 \Delta E_r.\end{aligned} \tag{6}$$

Proof It is well known that, if $A_r = 0$ or $\Delta A_r = 0$, then [1]

$$\begin{aligned}&(A_r + \Delta A_r)Q_1(A_r + \Delta A_r)^T \\ &\quad \leq A_r^T Q_1 A_r + \Delta A_r^T Q_1 \Delta A_r + A_r Q_1 \Delta A_r^T + \Delta A_r Q_1 A_r^T.\end{aligned}$$

If

$$\begin{aligned}A_r^T Q_1 \Delta A_r + A_r Q_1 \Delta A_r^T \leq \epsilon A_r^T Q_1 A_r \\ + \frac{1}{\epsilon}\Delta A_r Q_1 \Delta A_r^T,\end{aligned}$$

for any $\epsilon > 0$, then

$$(A_r + \Delta A_r)Q_1(A_r + \Delta A_r)^T \le (1+\epsilon)A_r^T Q_1 A_r + (1+\frac{1}{\epsilon})\Delta A_r^T Q_1 \Delta A_r.$$

And similarly if either $A_r, E_r = 0$ or $\Delta A_r, \Delta E_r = 0$, then

$$\begin{aligned}
&(A_r + \Delta A_r + E_r + \Delta E_r)Q_1(A_r \\
&\quad + \Delta A_r + E_r + \Delta E_r)^T \\
&\le A_r^T Q_1 A_r + \Delta A_r^T Q_1 \Delta A_r + A_r Q_1 \Delta A_r^T \\
&\quad + \Delta A_r Q_1 A_r^T + E_r^T Q_1 E_r + \Delta E_r^T Q_1 \Delta E_r \\
&\quad + E_r Q_1 \Delta E_r^T + \Delta E_r Q_1 E_r^T + \Delta A_r^T Q_1 A_r \\
&\quad + \Delta E_r^T Q_1 A_r^T + A_r Q_1 \Delta A_r^T + E_r Q_1 \Delta A_r^T \\
&\quad + \Delta E_r Q_1 \Delta A_r^T + A_r Q_1 E_r^T + \Delta A_r Q_1 E_r^T \\
&\quad + \Delta E_r Q_1 E_r^T + A_r Q_1 \Delta E_r^T + \Delta A_r Q_1 \Delta E_r^T \\
&\quad + E_r Q_1 \Delta E_r^T + E_r Q_1 A_r^T \\
&\quad \le A_r^T Q_1 A_r + \Delta A_r^T Q_1 \Delta A_r + E_r Q_1 E_r^T \\
&\quad + \Delta E_r^T Q_1 \Delta E_r + 3(\epsilon A_r Q_1 A_r^T + \frac{1}{\epsilon}\Delta A_r^T Q_1 \Delta A_r) \\
&\quad + 2\epsilon E_r Q_1 E_r^T + \frac{1}{\epsilon} E_r Q_1 E_r^T \\
&\quad + \epsilon \Delta A_r^T Q_1 \Delta A_r + \frac{2}{\epsilon}\Delta E_r^T Q_1 \Delta E_r.
\end{aligned}$$

□

Theorem 3.1 *Consider the neutral uncertain state delay system* (4)*, if Markovian switching at* $v(t) = 0$ *and* $w(t) = 0$ *such that*

$$\begin{aligned}
&d[x(t) - \widetilde{A}_{q_i}(t-q)] \\
&\quad = [\widetilde{A}_i x(t) + \widetilde{A}_{h_i} x(t-h)]\mathrm{dt} + [\widetilde{\mathrm{E}}_{\mathrm{i}}\mathrm{x(t)} + \widetilde{\mathrm{E}}_{\mathrm{h_i}}\mathrm{x(t-h)}]\mathrm{d}\omega \\
&\quad x_{t_0} = x(t_0+\theta) = \phi(\theta), \theta \in [-l, 0],\ r(0) = r_0, r(t) = i.
\end{aligned} \tag{7}$$

There exist matrices $P_i > 0, Q > 0, R > 0, W > 0$ *and*

$$\begin{aligned}
&\widetilde{A}_i P_i + \widetilde{A}_i^T P_i + \widetilde{E}_i P_i + \widetilde{E}_i^T P_i + \widetilde{A}_i^T (Q+R)\widetilde{A}_i \\
&\quad + \sum_{j=1}^{s} \Lambda_{ij} P_j + \widetilde{A}_{q_i} P_i + \widetilde{A}_{q_i}^T P_i + \widetilde{E}_{q_i} P_i + \widetilde{E}_{q_i}^T P_i \\
&\quad + \widetilde{A}_{q_i}^T (Q+R)\widetilde{A}_{q_i} - W + P_i(\widetilde{A}_{h_i} + \widetilde{E}_{h_i}) \\
&\quad - P_i(\widetilde{A}_{q_i} + \widetilde{E}_{q_i}) < 0, \\
&P_i(\widetilde{A}_{h_i} + \widetilde{E}_{h_i}) > 0, P_i(\widetilde{A}_{q_i} + \widetilde{E}_{q_i}) > 0, \\
&W = Q - \widetilde{A}_{q_i}^T (Q+R)\widetilde{A}_{q_i} > 0,
\end{aligned}$$

such that system (4) *is stochastically stable.*

Proof The Lyapunov functional of the above system is (see [8])

$$V(x(t),t,r(t)) = (x(t) - \widetilde{A}_{q_i}x(t-q))^T P_i(x(t) - \widetilde{A}_{q_i}x(t-q)) + \int_{t-q}^{t} x^T(\tau)Qx(\tau)\mathrm{d}\tau + \int_{t-h}^{t} x^T(\tau)Rx(\tau)\mathrm{d}\tau.$$

It can be derived by Ito's formula;

$$EV(x(t),t,r(t)) = EV(x(0),0,r(0)) + \int_0^s LV(x(s),s,r(s))\mathrm{d}s, \tag{8}$$

where $LV(x(s),s,r(s)) = \dot{V}(x(s),s,r(s)), L$ is the infinitesimal generator. Then

$$\frac{d}{\mathrm{dt}}V(x(t),t,r(t)) = \lim_{\Delta\to 0}\frac{1}{\Delta}[V(x(t+\Delta),t+\Delta, r(t+\Delta)=j) - V(x(t),t,r(t))],\ t,t+\Delta \in [0,\infty)$$

and

$$\begin{aligned} LV(x(t),t,r(t)) &= 2(x(t) - \widetilde{A}_{q_i}x(t-q))^T P_i(\dot{x}(t) - \widetilde{A}_{q_i}\dot{x}(t-q)) + x^T(t)Qx(t) - x^T(t-q)Qx(t-q) \\ &\quad + x^T(t)Rx(t) - x^T(t-h)Rx(t-h) = 2(x(t) - \widetilde{A}_{q_i}x(t-q))^T P_i(\widetilde{A}_i + \widetilde{E}_i + Q + R) \\ &\quad \times (x(t) - \widetilde{A}_{q_i}x(t-q)) + x^T(t)\sum_{j=1}^{s}\Lambda_{ij}P_jx(t) \\ &\quad + 2(x(t) - \widetilde{A}_{q_i}x(t-q))^T P_i(\widetilde{A}_{h_i}x(t-h) + \widetilde{E}_{h_i}x(t-h)) - 2x(t)^T P_i(\widetilde{A}_i + \widetilde{E}_i + Q + R) \\ &\quad \times \widetilde{A}_{q_i}x(t-q) - 2\widetilde{A}_{q_i}x(t-q)^T P_i(\widetilde{A}_i + \widetilde{E}_i + Q + R)x(t) - x^T(t-q)Wx(t-q) - x^T(t-h) \\ &\quad Rx(t-h) + 2(x(t) - \widetilde{A}_{q_i}x(t-q))P_i(\widetilde{A}_{h_i} + \widetilde{E}_{h_i})x^T(t-h). \end{aligned}$$

By Lemma 3.1 the above result can be rewritten as

$$
\begin{aligned}
LV(x(t),t,r(t)) \leq\ & x^T(t)[C_i\{A_i^T P_i A_i + \Delta A_i^T P_i \Delta A_i \\
& + E_i P_i E_i^T + \Delta E_i^T P_i \Delta E_i + 3(\epsilon A_i P_i A_i^T + \frac{1}{\epsilon}\Delta A_i^T P_i \Delta A_i) \\
& + 2\epsilon E_i P_i E_i^T + \frac{1}{\epsilon} E_i P_i E_i^T + \epsilon \Delta A_i^T P_i \Delta A_i + \frac{2}{\epsilon} \\
& \times \Delta E_i^T P_i \Delta E_i\} + C_i^{-1} P_i + P_i (Q+R)^T + P_i(Q+R) \\
& + \sum_{j=1}^{s} \Lambda_{ij} P_j] x(t) + x(t-q)^T [C_i\{A_{qi}^T P_i A_{qi} \\
& + \Delta A_{qi}^T P_i \Delta A_{qi} + E_{qi} P_i E_{qi}^T + \Delta E_{qi}^T P_i \Delta E_{qi} \\
& + 3(\epsilon A_{qi}^T P_i A_{qi}^T + \frac{1}{\epsilon}\Delta A_{qi}^T P_i \Delta A_{qi}^T) + 2\epsilon E_{qi} P_i E_{qi}^T \\
& + \frac{1}{\epsilon} E_{qi} P_i E_{qi}^T + \epsilon \Delta A_{qi}^T P_i \Delta A_{qi} + \frac{2}{\epsilon}\Delta E_{qi}^T P_i \Delta E_{qi}\} \\
& + C_i^{-1} P_i + P_i(Q+R)^T + P_i(Q+R) - W] \\
& \times x(t-q) - 2x^T(t)[C_i\{A_{qi}^T P_i A_{qi}^T \\
& + \Delta A_{qi}^T P_i \Delta A_{qi}^T + E_{qi} P_i E_{qi}^T + \Delta E_{qi}^T P_i \Delta E_{qi} \\
& + 3(\epsilon A_{qi}^T P_i A_{qi}^T + \frac{1}{\epsilon}\Delta A_{qi}^T P_i \Delta A_{qi}^T) + 2\epsilon E_{qi} P_i E_{qi}^T \\
& + \frac{1}{\epsilon} E_{qi} P_i E_{qi}^T + \epsilon \Delta A_{qi}^T P_i \Delta A_{qi} + \frac{2}{\epsilon}\Delta E_{qi}^T P_i \Delta E_{qi}\} \\
& + C_i^{-1} P_i + P_i(Q+R)^T + P_i(Q+R)] x(t-q) \\
& + 2x^T(t-q)[C_i\{A_{qi}^T P_i A_{qi}^T + \Delta A_{qi}^T P_i \Delta A_{qi}^T \\
& + E_{qi} P_i E_{qi}^T + \Delta E_{qi}^T P_i \Delta E_{qi} + 3(\epsilon A_{qi}^T P_i A_{qi}^T \\
& + \frac{1}{\epsilon}\Delta A_{qi}^T P_i \Delta A_{qi}^T) + 2\epsilon E_{qi} P_i E_{qi}^T + \frac{1}{\epsilon} E_{qi} P_i E_{qi}^T \\
& + \epsilon \Delta A_{qi}^T P_i \Delta A_{qi} + \frac{2}{\epsilon}\Delta E_{qi}^T P_i \Delta E_{qi}\} + C_i^{-1} P_i \\
& + P_i(Q+R)^T + P_i(Q+R)] x(t) + x^T(t)[(1+\epsilon) \\
& \times A_{h_i}^T P_i A_{h_i} + (1+\frac{1}{\epsilon})\Delta A_{h_i}^T P_i \Delta A_{h_i}] x(t-h) \\
& - x^T(t-q)[(1+\epsilon) A_{q_i}^T P_i A_{q_i} + (1+\frac{1}{\epsilon})\Delta A_{q_i}^T P_i \Delta A_{q_i}] \\
& \times x(t-h) - x^T(t-h)[\frac{1}{2}(C_{hi} + C_{hi}^{-1})] R x(t-h) \\
& + x(t)[(1+\epsilon) A_{h_i}^T P_i A_{h_i} + (1+\frac{1}{\epsilon})\Delta A_{h_i}^T P_i \Delta A_{h_i}] \\
& \times x(t-h)^T - x^T(t-q)[(1+\epsilon) A_{q_i}^T P_i A_{q_i} \\
& + (1+\frac{1}{\epsilon})\Delta A_{q_i}^T P_i \Delta A_{q_i}] x^T(t-h) \\
=\ & \eta(t)^T \Xi_i \eta(t),
\end{aligned} \tag{9}
$$

where

$$\begin{aligned}
\Xi_i &= \begin{pmatrix} \Omega_{11} & \Omega_{12}^T & \Omega_{13} \\ \Omega_{21} & \Omega_{22} & \Omega_{23}^T \\ \Omega_{31}^T & \Omega_{32} & \Omega_{33} \end{pmatrix}, \\
\Omega_{11} &= C_i\{A_i^T P_i A_i + \Delta A_i^T P_i \Delta A_i + E_i P_i E_i^T \\
&\quad + \Delta E_i^T P_i \Delta E_i + 3(\epsilon A_i P_i A_i^T + \frac{1}{\epsilon}\Delta A_i^T P_i \Delta A_i) \\
&\quad + 2\epsilon E_i P_i E_i^T + \frac{1}{\epsilon} E_i P_i E_i^T + \epsilon \Delta A_i^T P_i \Delta A_i \\
&\quad + \frac{2}{\epsilon}\Delta E_i^T P_i \Delta E_i\} + C_i^{-1} P_i + P_i (Q+R)^T \\
&\quad + P_i(Q+R) + \sum_{j=1}^{s} \Lambda_{ij} P_j \\
\Omega_{22} &= C_i\{A_{qi}^T P_i A_{qi} + \Delta A_{qi}^T P_i \Delta A_{qi} + E_{qi} P_i E_{qi}^T \\
&\quad + \Delta E_{qi}^T P_i \Delta E_{qi} + 3(\epsilon A_{qi}^T P_i A_{qi}^T + \frac{1}{\epsilon}\Delta A_{qi}^T \\
&\quad \times P_i \Delta A_{qi}^T) + 2\epsilon E_{qi} P_i E_{qi}^T + \frac{1}{\epsilon} E_{qi} P_i E_{qi}^T \\
&\quad + \epsilon \Delta A_{qi}^T P_i \Delta A_{qi} + \frac{2}{\epsilon}\Delta E_{qi}^T P_i \Delta E_{qi}\} + C_i^{-1} P_i \\
&\quad + P_i(Q+R)^T + P_i(Q+R) - W \\
\Omega_{21} &= -2(C_i\{A_{qi}^T P_i A_{qi}^T + \Delta A_{qi}^T P_i \Delta A_{qi}^T \\
&\quad + E_{qi} P_i E_{qi}^T + \Delta E_{qi}^T P_i \Delta E_{qi} + 3(\epsilon A_{qi}^T P_i A_{qi}^T \\
&\quad + \frac{1}{\epsilon}\Delta A_{qi}^T P_i \Delta A_{qi}^T) + 2\epsilon E_{qi} P_i E_{qi}^T \\
&\quad + \frac{1}{\epsilon} E_{qi} P_i E_{qi}^T + \epsilon \Delta A_{qi}^T P_i \Delta A_{qi} \\
&\quad + \frac{2}{\epsilon}\Delta E_{qi}^T P_i \Delta E_{qi}\} + C_i^{-1} P_i \\
&\quad + P_i(Q+R)^T + P_i(Q+R)), \\
\Omega_{23} &= -(1+\epsilon) A_{q_i}^T P_i A_{q_i} - (1+\frac{1}{\epsilon})\Delta A_{q_i}^T P_i \Delta A_{q_i}, \\
\Omega_{31} &= (1+\epsilon) A_{h_i}^T P_i A_{h_i} + (1+\frac{1}{\epsilon})\Delta A_{h_i}^T P_i \Delta A_{h_i}, \\
\Omega_{33} &= -\frac{1}{2}(C_{hi} + C_{hi}^{-1}) R
\end{aligned}$$

and $\eta(t) = [x(t), x(t-q), x(t-h)]$. Based on the inequality (9), mean-square stability of the system (3)–(4) can be proved if

$$\frac{LV(x(t), t, r(t))}{V(x(t), t, r(t))} = \frac{\eta(t)^T \Xi_i \eta(t)}{\Lambda} \tag{10}$$

where $\Lambda = (x(t) - A_{q_i} x(t-q))^T P_i (x(t) - A_{q_i} x(t-q)) + \int_{t-q}^{t} x^T(\tau) Q x(\tau) \mathrm{d}\tau + \int_{\mathrm{t-h}}^{\mathrm{t}} \mathrm{x}^{\mathrm{T}}(\tau) \mathrm{R} \mathrm{x}(\tau) \mathrm{d}\tau$.

Note that, if $\Xi_i < 0$, $P_i > 0$ and $x \neq 0$, such that

$$\begin{aligned}&\frac{LV(x(t),t,r(t))}{V(x(t),t,r(t))}\\&\quad\leq -\min_{i\in S}\left\{\frac{\lambda_{\min}(-\Xi_i)}{\lambda_{\max}(P_i)+q\lambda_{\max}(Q)+h\lambda_{\max}(R)}\right\}.\end{aligned}\tag{11}$$

If

$$\beta=\min_{i\in S}\left\{\frac{\lambda_{\min}(-\Xi_i)}{\lambda_{\max}(P_i)+q\lambda_{\max}(Q)+h\lambda_{\max}(R)}\right\},$$

and $\beta>0$, then

$$LV(x(t),t,r(t))\leq -\beta V(x(t),t,r(t)).$$

Therefore by Ito's formula

$$\begin{aligned}&\kappa\{V(x(t),t,r(t))-V(x(0),0,r(0))\}\\&\quad=E\int_0^t LV(x(s),s,r(s))\mathrm{ds}\\&\quad\leq-\beta\int_0^t EV(x(s),s,r(s))\mathrm{ds}.\end{aligned}$$

By Gronwell-Bellman lemma the above result can be rewritten as

$$E\{V(x(t),t,r(t))\leq exp(-\beta t)V(x(0),0,r(0))\},$$

for all $Q>0, R>0$. As on Lemma 3.1, the following inequalities are holds

$$E\int_{t-q}^t x^T(\tau)Qx(\tau)\mathrm{d}\tau>0, \mathrm{E}\int_{\mathrm{t-h}}^{\mathrm{t}}\mathrm{x}^{\mathrm{T}}(\tau)\mathrm{Rx}(\tau)\mathrm{d}\tau>0,$$

and

$$\begin{aligned}&E\{(x(t)-A_{q_i}x(t-q))^TP_i(x(t)-A_{q_i}x(t-q))\}\\&\quad=E\{V(x(t),t,r(t))\}-E\int_{t-q}^t x^T(\tau)Qx(\tau)\mathrm{d}\tau\\&\qquad+E\int_{t-h}^t x^T(\tau)Rx(\tau)\mathrm{d}\tau\\&\qquad\leq exp(-\beta t)V(x(0),0,r(0)),\end{aligned}$$

for all $r(0) \in S$. Let

$$E\{\int_0^T (x(t) - A_{q_i}x(t-q))^T P_i(x(t) - A_{q_i}x(t-q))\mathrm{dt}\}$$
$$< \int_0^T exp(-\beta T)\mathrm{dtV(x(0), 0, r(0))}$$
$$= \frac{-1}{\beta}[exp(-\beta T) - 1]V(x(0), 0, r(0)).$$

Taking the limit as $T \to \infty$, for all $T \in [0, \infty)$,

$$\lim_{T\to\infty} E\{\int_0^T (x(t) - A_{q_i}x(t-q))^T P_i(x(t) - A_{q_i}x(t-q))\mathrm{dt}\}$$
$$\leq \frac{1}{\beta}x_0^T(\lambda_{\max}(P_i) + q\lambda_{\max}(Q) + h\lambda_{\max}(T))x_0.$$

If $P_i > 0, i \in S$, then

$$\lim_{T\to\infty} E\{\int_0^T (x(t) - A_{q_i}x(t-q))^T P_i(x(t) - A_{q_i}x(t-q))\mathrm{dt}\Big|\phi, \mathrm{r}_0\}$$
$$\leq x_0^T\overline{P}x_0,$$

where

$$\overline{P} = \max_{i\in S}\left\{\frac{\lambda_{\max}(P_i) + q\lambda_{\max}(Q) + h\lambda_{\max}(R)}{\beta\lambda_{\min}(P_i)}\right\},$$

this implies that closed-loop system (4) under the control law (3) is stochastically stable.

Form Schur complement, $\Xi_i < 0, i = 1, \ldots, s$, if and only if

$$\begin{aligned}
&C_i\{A_i^T P_i A_i + \Delta A_i^T P_i \Delta A_i + E_i P_i E_i^T + \Delta E_i^T P_i \Delta E_i \\
&\quad + 3(\epsilon A_i P_i A_i^T + \frac{1}{\epsilon}\Delta A_i^T P_i \Delta A_i) + 2\epsilon E_i P_i E_i^T \\
&\quad + \frac{1}{\epsilon} E_i P_i E_i^T + \epsilon \Delta A_i^T P_i \Delta A_i + \frac{2}{\epsilon}\Delta E_i^T P_i \Delta E_i\} \\
&\quad + C_i^{-1} P_i + P_i(Q+R)^T + P_i(Q+R) + \sum_{j=1}^{s} \Lambda_{ij} P_j \\
&\quad + (2C_i\{A_{qi}^T P_i A_{qi} + \Delta A_{qi}^T P_i \Delta A_{qi} + E_{qi} P_i E_{qi}^T \\
&\quad + \Delta E_{qi}^T P_i \Delta E_{qi} + 3(\epsilon A_{qi}^T P_i A_{qi}^T + \frac{1}{\epsilon}\Delta A_{qi}^T P_i \Delta A_{qi}^T) \\
&\quad + 2\epsilon E_{qi} P_i E_{qi}^T + \frac{1}{\epsilon} E_{qi} P_i E_{qi}^T + \epsilon \Delta A_{qi}^T P_i \Delta A_{qi} \\
&\quad + \frac{2}{\epsilon}\Delta E_{qi}^T P_i \Delta E_{qi}\} + C_i^{-1} P_i + P_i(Q+R)^T \\
&\quad + P_i(Q+R))^T (C_i\{A_{qi}^T P_i A_{qi}^T + \Delta A_{qi}^T P_i \Delta A_{qi}^T \\
&\quad + E_{qi} P_i E_{qi}^T + \Delta E_{qi}^T P_i \Delta E_{qi} + 3(\epsilon A_{qi}^T P_i A_{qi}^T \\
&\quad + \frac{1}{\epsilon}\Delta A_{qi}^T P_i \Delta A_{qi}^T) + 2\epsilon E_{qi} P_i E_{qi}^T + \frac{1}{\epsilon} E_{qi} P_i E_{qi}^T \\
&\quad + \epsilon \Delta A_{qi}^T P_i \Delta A_{qi} + \frac{2}{\epsilon}\Delta E_{qi}^T P_i \Delta E_{qi}\} + C_i^{-1} P_i \\
&\quad + P_i(Q+R)^T + P_i(Q+R) - W)^{-1}(2(C_i\{A_{qi}^T \\
&\quad \times P_i A_{qi}^T + \Delta A_{qi}^T P_i \Delta A_{qi}^T + E_{qi} P_i E_{qi}^T \\
&\quad + \Delta E_{qi}^T P_i \Delta E_{qi} + 3(\epsilon A_{qi}^T P_i A_{qi}^T + \frac{1}{\epsilon}\Delta A_{qi}^T P_i \Delta A_{qi}^T) \\
&\quad + 2\epsilon E_{qi} P_i E_{qi}^T + \frac{1}{\epsilon} E_{qi} P_i E_{qi}^T + \epsilon \Delta A_{qi}^T P_i \Delta A_{qi} \\
&\quad + \frac{2}{\epsilon}\Delta E_{qi}^T P_i \Delta E_{qi}\} + C_i^{-1} P_i + P_i(Q+R)^T \\
&\quad + P_i(Q+R)) + ((1+\epsilon)A_{q_i}^T P_i A_{q_i} - (1+\frac{1}{\epsilon}) \\
&\quad \times \Delta A_{q_i}^T P_i \Delta A_{q_i})^T I((1+\epsilon)A_{q_i}^T P_i A_{q_i} - (1+\frac{1}{\epsilon}) \\
&\quad \times \Delta A_{q_i}^T P_i \Delta A_{q_i}) + ((1+\epsilon)A_{h_i}^T P_i A_{h_i} + (1+\frac{1}{\epsilon}) \\
&\quad \times \Delta A_{h_i}^T P_i \Delta A_{h_i})^T (\frac{1}{2}(C_{hi} + C_{hi}^{-1})R)^{-1} \\
&\quad \times ((1+\epsilon)A_{h_i}^T P_i A_{h_i} + (1+\frac{1}{\epsilon})\Delta A_{h_i}^T P_i \Delta A_{h_i}) < 0.
\end{aligned} \tag{12}$$

If $X_i = P_i^{-1}$, then multiplying P_i^{-1} by (12), we get

$$
\begin{aligned}
&C_iX_i\{A_i^TP_iA_i+\Delta A_i^TP_i\Delta A_i+E_iP_iE_i^T\\
&\quad+\Delta E_i^TP_i\Delta E_i+3(\epsilon A_iP_iA_i^T+\frac{1}{\epsilon}\Delta A_i^TP_i\Delta A_i)\\
&\quad+2\epsilon E_iP_iE_i^T+\frac{1}{\epsilon}E_iP_iE_i^T+\epsilon\Delta A_i^TP_i\Delta A_i\\
&\quad+\frac{2}{\epsilon}\Delta E_i^TP_i\Delta E_i\}+C_i^{-1}P_i+P_i(Q+R)^T\\
&\quad+P_i(Q+R)+\sum_{j=1}^{s}\Lambda_{ij}P_j+(2C_iX_i\{A_{qi}^TP_iA_{qi}^T\\
&\quad+\Delta A_{qi}^TP_i\Delta A_{qi}^T+E_{qi}P_iE_{qi}^T+\Delta E_{qi}^TP_i\Delta E_{qi}\\
&\quad+3(\epsilon A_{qi}^TP_iA_{qi}+\frac{1}{\epsilon}\Delta A_{qi}^TP_i\Delta A_{qi})\\
&\quad+2\epsilon E_{qi}P_iE_{qi}^T+\frac{1}{\epsilon}E_{qi}P_iE_{qi}^T+\epsilon\Delta A_{qi}^TP_i\Delta A_{qi}\\
&\quad+\frac{2}{\epsilon}\Delta E_{qi}^TP_i\Delta E_{qi}\}+C_i^{-1}X_iP_i+X_iP_i(Q+R)^T\\
&\quad+X_iP_i(Q+R))^T(C_i\{A_{qi}^TP_iA_{qi}^T+\Delta A_{qi}^TP_i\Delta A_{qi}^T\\
&\quad+E_{qi}P_iE_{qi}^T+\Delta E_{qi}^TP_i\Delta E_{qi}+3(\epsilon A_{qi}^TP_iA_{qi}^T\\
&\quad+\frac{1}{\epsilon}\Delta A_{qi}^TP_i\Delta A_{qi}^T)+2\epsilon E_{qi}P_iE_{qi}^T+\frac{1}{\epsilon}E_{qi}P_iE_{qi}^T\\
&\quad+\epsilon\Delta A_{qi}^TP_i\Delta A_{qi}+\frac{2}{\epsilon}\Delta E_{qi}^TP_i\Delta E_{qi}\}+C_i^{-1}P_i\\
&\quad+P_i(Q+R)^T+P_i(Q+R)-W)^{-1}\\
&\quad\times(2(C_i\{A_{qi}^TP_iA_{qi}^T+\Delta A_{qi}^TP_i\Delta A_{qi}^T+E_{qi}P_iE_{qi}^T\\
&\quad+\Delta E_{qi}^TP_i\Delta E_{qi}+3(\epsilon A_{qi}^TP_iA_{qi}^T+\frac{1}{\epsilon}\Delta A_{qi}^TP_i\Delta A_{qi}^T)\\
&\quad+2\epsilon E_{qi}P_iE_{qi}^T+\frac{1}{\epsilon}E_{qi}P_iE_{qi}^T+\epsilon\Delta A_{qi}^TP_i\Delta A_{qi}\\
&\quad+\frac{2}{\epsilon}\Delta E_{qi}^TP_i\Delta E_{qi}\}+C_i^{-1}P_i+P_i(Q+R)^T\\
&\quad+P_i(Q+R))+((1+\epsilon)A_{q_i}^TP_iA_{q_i}-(1+\frac{1}{\epsilon})\\
&\quad\times\Delta A_{q_i}^TP_i\Delta A_{q_i})^TX_i((1+\epsilon)A_{q_i}^TP_iA_{q_i}\\
&\quad-(1+\frac{1}{\epsilon})\Delta A_{q_i}^TP_i\Delta A_{q_i})+((1+\epsilon)A_{h_i}^TP_iA_{h_i}\\
&\quad+(1+\frac{1}{\epsilon})\Delta A_{h_i}^TP_i\Delta A_{h_i})^T(\frac{1}{2}(C_{hi}+C_{hi}^{-1})R)^{-1}X_i\\
&\quad\times((1+\epsilon)A_{h_i}^TP_iA_{h_i}+(1+\frac{1}{\epsilon})\Delta A_{h_i}^TP_i\Delta A_{h_i})<0.
\end{aligned}
\tag{13}
$$

As on LMI, the above expression can be rewritten as

$$
\begin{pmatrix}
\Gamma_i & \Theta_i & \Psi_{1i} & \Psi_{2i}\\
\Theta_i^T & -\Phi_i & 0 & 0\\
\Psi_{1i}^T & 0 & -\mu_i & 0\\
\Psi_{2i}^T & 0 & 0 & -I
\end{pmatrix}<0,
\tag{14}
$$

where

$$
\begin{aligned}
\Gamma_i &= \begin{pmatrix} \chi_i & \chi_{i1} \\ \chi_{i2} & \chi_{i3} \end{pmatrix} > 0, \\
\chi_i &= C_i\{A_i^T X_i A_i + \Delta A_i^T X_i \Delta A_i + E_i X_i E_i^T \\
&\quad + \Delta E_i^T X_i \Delta E_i + 3(\epsilon A_i X_i A_i^T + \frac{1}{\epsilon}\Delta A_i^T X_i \Delta A_i) \\
&\quad + 2\epsilon E_i X_i E_i^T + \frac{1}{\epsilon} E_i X_i E_i^T + \epsilon \Delta A_i^T X_i \Delta A_i \\
&\quad + \frac{2}{\epsilon}\Delta E_i^T X_i \Delta E_i\} + C_i^{-1} X_i + X_i(Q+R)^T \\
&\quad + X_i(Q+R) + \Lambda_{ii} X_i, \\
\chi_{i1} &= ((1+\epsilon)A_{h_i}^T X_i A_{h_i} + (1+\frac{1}{\epsilon})\Delta A_{h_i}^T X_i \Delta A_{h_i})^T \\
\chi_{i2} &= ((1+\epsilon)A_{h_i}^T X_i A_{h_i} + (1+\frac{1}{\epsilon})\Delta A_{h_i}^T X_i \Delta A_{h_i}) \\
\chi_{i3} &= -\frac{1}{2}(C_{hi} + C_{hi}^{-1})R \\
\Theta_i &= 2C_i\{A_{qi}^T X_i A_{qi} + \Delta A_{qi}^T X_i \Delta A_{qi} + E_{qi} \\
&\quad \times X_i E_{qi}^T + \Delta E_{qi}^T X_i \Delta E_{qi} + 3(\epsilon A_{qi}^T X_i A_{qi}^T \\
&\quad + \frac{1}{\epsilon}\Delta A_{qi}^T X_i \Delta A_{qi}^T) + 2\epsilon E_{qi} X_i E_{qi}^T \\
&\quad + \frac{1}{\epsilon} E_{qi} X_i E_{qi}^T + \epsilon \Delta A_{qi}^T X_i \Delta A_{qi} \\
&\quad + \frac{2}{\epsilon}\Delta E_{qi}^T X_i \Delta E_{qi}\} + X_i^{-1} C_i^{-1} X_i \\
&\quad + X_i(Q+R)^T + X_i(Q+R), \\
\Phi_i &= C_i\{A_{qi}^T X_i A_{qi}^T + \Delta A_{qi}^T X_i \Delta A_{qi}^T \\
&\quad + E_{qi} X_i E_{qi}^T + \Delta E_{qi}^T X_i \Delta E_{qi} + 3(\epsilon A_{qi}^T X_i A_{qi}^T \\
&\quad + \frac{1}{\epsilon}\Delta A_{qi}^T X_i \Delta A_{qi}^T) + 2\epsilon E_{qi} X_i E_{qi}^T \\
&\quad + \frac{1}{\epsilon} E_{qi} X_i E_{qi}^T + \epsilon \Delta A_{qi}^T X_i \Delta A_{qi} \\
&\quad + \frac{2}{\epsilon}\Delta E_{qi}^T X_i \Delta E_{qi}\} + X_i^{-1} C_i^{-1} X_i \\
&\quad + X_i(Q+R)^T + X_i(Q+R) - X_i^{-1} W X_i, \\
\Psi_{1i} &= \sqrt{\Lambda_{i1}} X_i, \sqrt{\Lambda_{i2}} X_i, \cdots, \sqrt{\Lambda_{iN}} X_i, \\
\mu_i &= \mathrm{diag}(X_1, X_2, \cdots, X_{i-1}, X_i, X_{i+1}, X_N) \\
\Psi_{2i} &= (1+\epsilon)A_{q_i}^T X_i A_{q_i} - (1+\frac{1}{\epsilon})\Delta A_{q_i}^T X_i \Delta A_{q_i}.
\end{aligned}
$$

□

Theorem 3.2 *The system* (4) *is stochastically stabilizable, if* $R > 0, Q > 0, W > 0, X_i > 0, P_i > 0,\ Y_i, i = 1, 2, \cdots, s$ *exists and the disturbance* $w(t) = 0$, *then controller* $K_i = Y_i X_i^{-1}$ *is exists such that the following inequality holds*

$$
\begin{aligned}
&(\widetilde{A}_i + \widetilde{B}_i)P_i + (\widetilde{A}_i + \widetilde{B}_i)^T P_i + \widetilde{E}_i P_i + \widetilde{E}_i^T P_i \\
&\quad + \widetilde{A}_i^T (Q + R)\widetilde{A}_i + \sum_{j=1}^{s} \Lambda_{ij} P_j + \widetilde{A}_{q_i} P_i + \widetilde{A}_{q_i}^T P_i \\
&\quad + \widetilde{E}_{q_i} P_i + \widetilde{E}_{q_i}^T P_i + \widetilde{A}_{q_i}^T (Q + R)\widetilde{A}_{q_i} - W \\
&\quad + P_i(\widetilde{A}_{h_i} + \widetilde{E}_{h_i}) - P_i(\widetilde{A}_{q_i} + \widetilde{E}_{q_i}) < 0, \\
&P_i(\widetilde{A}_{h_i} + \widetilde{E}_{h_i}) > 0, P_i(\widetilde{A}_{q_i} + \widetilde{E}_{q_i}) > 0, \\
&W = Q - \widetilde{A}_{q_i}^T (Q + R)\widetilde{A}_{q_i} > 0.
\end{aligned}
$$

Proof It follows that Theorem 3.1, Ξ_i can be written as

$$
\begin{aligned}
\Xi_i &= \begin{pmatrix} \Omega_{11} & \Omega_{12}^T & \Omega_{13} \\ \Omega_{21} & \Omega_{22} & \Omega_{23}^T \\ \Omega_{31}^T & \Omega_{32} & \Omega_{33} \end{pmatrix}, \\
\Omega_{11} &= C_i\{(A_i + B_{1i}K_i)^T P_i (A_i + B_{1i}K_i) + \Delta(A_i + B_{1i}K_i)^T P_i \Delta(A_i + B_{1i}K_i) \\
&\quad + E_i P_i E_i^T + \Delta E_i^T P_i \Delta E_i + 3(\epsilon(A_i + B_{1i}K_i)P_i(A_i + B_{1i}K_i)^T + \frac{1}{\epsilon}\Delta(A_i \\
&\quad + B_{1i}K_i)^T P_i \Delta(A_i + B_{1i}K_i) + 2\epsilon E_i P_i E_i^T + \frac{1}{\epsilon} E_i P_i E_i^T + \epsilon(\Delta(A_i + B_{1i}K_i))^T P_i \\
&\quad \times \Delta(A_i + B_{1i}K_i) + \frac{2}{\epsilon}\Delta E_i^T P_i \Delta E_i\} + C_i^{-1} P_i + P_i(Q + R)^T + P_i(Q + R) + \sum_{j=1}^{s} \Lambda_{ij} P_j \\
\Omega_{22} &= C_i\{(A_{qi} + B_{1q_i}K_i)^T P_i (A_{qi} + B_{1q_i}K_i) + (\Delta(A_{qi} + B_{1q_i}K_i))^T P_i \Delta(A_{qi} + B_{1q_i}K_i) \\
&\quad + E_{qi} P_i E_{qi}^T + \Delta E_{qi}^T P_i \Delta E_{qi} + 3(\epsilon(A_{qi} + B_{1q_i}K_i)^T P_i (A_{qi} + B_{1q_i}K_i) + \frac{1}{\epsilon}(\Delta(A_{qi} \\
&\quad + B_{1q_i}K_i))^T P_i \Delta(A_{qi} + B_{1q_i}K_i)) + 2\epsilon E_{qi} \times P_i E_{qi}^T + \frac{1}{\epsilon} E_{qi} P_i E_{qi}^T + \epsilon(\Delta(A_{qi} + B_{1q_i}K_i))^T \\
&\quad \times P_i \Delta(A_{qi} + B_{1q_i}K_i) + \frac{2}{\epsilon}\Delta E_{qi}^T P_i \Delta E_{qi}\} + C_i^{-1} P_i + P_i(Q + R)^T + P_i(Q + R) - W \\
\Omega_{21} &= -2(C_i\{(A_{qi} + B_{1q_i}K_i)^T P_i (A_{qi} + B_{1q_i}K_i) + (\Delta(A_{qi} + B_{1q_i}K_i))^T P_i \Delta(A_{qi} + B_{1q_i}K_i) \\
&\quad + E_{qi} P_i E_{qi}^T + \Delta E_{qi}^T P_i \Delta E_{qi} + 3(\epsilon(A_{qi} + B_{1q_i}K_i)^T P_i (A_{qi} + B_{1q_i}K_i) + \frac{1}{\epsilon}(\Delta(A_{qi} \\
&\quad + B_{1q_i}K_i))^T P_i \Delta(A_{qi} + B_{1q_i}K_i) + 2\epsilon E_{qi} \times P_i E_{qi}^T + \frac{1}{\epsilon} E_{qi} P_i E_{qi}^T + \epsilon(\Delta(A_{qi} + B_{1q_i}K_i))^T \\
&\quad \times P_i \Delta(A_{qi} + B_{1q_i}K_i) + \frac{2}{\epsilon}\Delta E_{qi}^T P_i \Delta E_{qi}\} + C_i^{-1} P_i + P_i(Q + R)^T + P_i(Q + R)), \\
\Omega_{23} &= -(1 + \epsilon)(A_{qi} + B_{1q_i}K_i)^T P_i (A_{qi} + B_{1q_i}K_i) - (1 + \frac{1}{\epsilon})(\Delta(A_{qi} \\
&\qquad + B_{1q_i}K_i))^T P_i \Delta(A_{qi} + B_{1q_i}K_i), \\
\Omega_{31} &= (1 + \epsilon)(A_{h_i} + B_{1h_i}K_i)^T P_i (A_{h_i} + B_{1h_i}K_i) \\
&\quad + (1 + \frac{1}{\epsilon})(\Delta(A_{h_i} + B_{1h_i}K_i))^T P_i \Delta(A_{h_i} + B_{1h_i}K_i), \\
\Omega_{33} &= -\frac{1}{2}(C_{hi} + C_{hi}^{-1})R.
\end{aligned}
$$

It is noting that above inequality and Theorem 3.1 plays an important role in proving stability of H^{∞} control system. □

The robust H_∞ control

Theorem 4.1 *Given a scalar $\gamma > 0, Y_i, i = 1, 2, \ldots, s$ and $K_i = Y_i X_i^{-1}$ are exist in H_∞ control, there exist positive definite matrices $P_i > 0, R > 0, Q > 0, E_i > 0, W > 0$ such that system (1)–(3) is roust stochastically stable with given disturbance attenuation level γ, if the following inequality holds*

$$\begin{aligned}
&(\widetilde{A}_i + \widetilde{B}_i)P_i + (\widetilde{A}_i + \widetilde{B}_i)^T P_i + \widetilde{E}_i P_i + \widetilde{E}_i^T P_i \\
&\quad + \widetilde{A}_i^T (Q + R)\widetilde{A}_i + \sum_{j=1}^{s} \Lambda_{ij} P_j + \widetilde{A}_{q_i} P_i + \widetilde{A}_{q_i}^T P_i \\
&\quad + \widetilde{E}_{q_i} P_i + \widetilde{E}_{q_i}^T P_i + \widetilde{A}_{q_i}^T (Q + R)\widetilde{A}_{q_i} - W \\
&\quad + P_i(\widetilde{A}_{h_i} + \widetilde{E}_{h_i}) - P_i(\widetilde{A}_{q_i} + \widetilde{E}_{q_i}) - \gamma^2 I < 0, \\
&\ P_i(\widetilde{A}_{h_i} + \widetilde{E}_{h_i}) > 0,\ P_i(\widetilde{A}_{q_i} + \widetilde{E}_{q_i}) > 0
\end{aligned}$$

and

$$W = Q - \widetilde{A}_{q_i}^T (Q + R)\widetilde{A}_{q_i} > 0.$$

Proof As on Theorem 3.1 the system (4) with $q \in L^2([-l, 0] : R^n)$ exists in H_∞, to prove H_∞ constraint of system (4) is mean-square stochastically stable, we need Lyapunov functional $V(x(t), t, r(t))$ and

$$J(t) = E\{\int_0^t [z^T(s)z(s) - \gamma^2 v^T(s)v(s)]\mathrm{d}s\}. \tag{15}$$

It follows from Dynkin's formula and fact that $x(0) = 0$, then

$$E\{V(x(t), t, r(t)\} = E\{\int_0^t LV(x(s), s, r(s))\mathrm{d}s\}. \tag{16}$$

Substitute (16) in (15), we get

$$\begin{aligned}
J(t) &= E\{\int_0^t [z^T(s)z(s) - \gamma^2 v^T(s)v(s) \\
&\quad + LV(x(s), s, r(s))\mathrm{d}s\} - \mathrm{EV(x(t), t, r(t))} \\
&\le E\{\int_0^t [z^T(s)z(s) - \gamma^2 v^T(s)v(s) \\
&\quad + LV(x(s), s, r(s))\mathrm{d}s\}.
\end{aligned} \tag{17}$$

Now let

$$\xi^T(t) = [x^T(t), x^T(t - q), x^T(t - h), w^T(t)], \tag{18}$$

Substitute (18) in (17), we get

$$\begin{aligned}
&z^T(s)z(s) - \gamma^2 w^T(s)w(s) + LV(x(s), s, r(s)) \\
&\quad \le \xi^T(s)\Gamma_i \xi(s),
\end{aligned} \tag{19}$$

where

$$\Gamma_i = \begin{pmatrix} \psi_{11} & \psi_{12}^T & \psi_{13} & 0 & \psi_{15} \\ \psi_{21} & \psi_{22} & \psi_{23}^T & \psi_{24} & 0 \\ \psi_{31}^T & \psi_{32} & \psi_{33} & 0 & 0 \\ 0 & \psi_{42}^T & 0 & \psi_{44} & 0 \\ \psi_{51}^T & 0 & 0 & 0 & \psi_{55} \end{pmatrix},$$

and

$$\begin{aligned}
\psi_{11} = {} & C_i\{(A_i + B_{1i}K_i)^T P_i(A_i + B_{1i}K_i) + \Delta(A_i + B_{1i}K_i)^T P_i \Delta(A_i + B_{1i}K_i) \\
& + E_i P_i E_i^T + \Delta E_i^T P_i \Delta E_i + 3(\epsilon(A_i + B_{1i}K_i) \times P_i(A_i + B_{1i}K_i)^T + \frac{1}{\epsilon}\Delta(A_i + B_{1i}K_i)^T \\
& \times P_i \Delta(A_i + B_{1i}K_i) + 2\epsilon E_i P_i E_i^T + \frac{1}{\epsilon} E_i P_i E_i^T + \epsilon(\Delta(A_i + B_{1i}K_i))^T P_i \Delta(A_i + B_{1i}K_i) \\
& + \frac{2}{\epsilon}\Delta E_i^T P_i \Delta E_i\} + C_i^{-1} P_i + P_i(Q+R)^T + P_i(Q+R) + \sum_{j=1}^{s} \Lambda_{ij} P_j \\
\psi_{22} = {} & C_i\{(A_{qi} + B_{1q_i}K_i)^T P_i(A_{qi} + B_{1q_i}K_i) + (\Delta(A_{qi} + B_{1q_i}K_i))^T P_i \Delta(A_{qi} + B_{1q_i}K_i) \\
& + E_{qi} P_i E_{qi}^T + \Delta E_{qi}^T P_i \Delta E_{qi} + 3(\epsilon(A_{qi} + B_{1q_i}K_i)^T P_i(A_{qi} + B_{1q_i}K_i) + \frac{1}{\epsilon}(\Delta(A_{qi} \\
& + B_{1q_i}K_i))^T P_i \Delta(A_{qi} + B_{1q_i}K_i)) + 2\epsilon E_{qi} P_i E_{qi}^T + \frac{1}{\epsilon} E_{qi} P_i E_{qi}^T + \epsilon(\Delta(A_{qi} \\
& + B_{1q_i}K_i))^T P_i \Delta(A_{qi} + B_{1q_i}K_i) + \frac{2}{\epsilon}\Delta E_{qi}^T \times P_i \Delta E_{qi}\} + C_i^{-1} P_i \\
& + P_i(Q+R)^T + P_i(Q+R) - W \\
\psi_{21} = {} & -2(C_i\{(A_{qi} + B_{1q_i}K_i)^T P_i(A_{qi} + B_{1q_i}K_i) + (\Delta(A_{qi} + B_{1q_i}K_i))^T P_i \Delta(A_{qi} + B_{1q_i}K_i) \\
& + E_{qi} P_i E_{qi}^T + \Delta E_{qi}^T P_i \Delta E_{qi} + 3(\epsilon(A_{qi} + B_{1q_i}K_i)^T P_i(A_{qi} + B_{1q_i}K_i) + \frac{1}{\epsilon}(\Delta(A_{qi} \\
& + B_{1q_i}K_i))^T P_i \Delta(A_{qi} + B_{1q_i}K_i) + 2\epsilon E_{qi} P_i E_{qi}^T + \frac{1}{\epsilon} E_{qi} P_i E_{qi}^T + \epsilon(\Delta(A_{qi} + B_{1q_i}K_i))^T P_i \\
& \times \Delta(A_{qi} + B_{1q_i}K_i) + \frac{2}{\epsilon}\Delta E_{qi}^T P_i \Delta E_{qi}\} + C_i^{-1} P_i + P_i(Q+R)^T + P_i(Q+R)), \\
\psi_{23} = {} & -(1+\epsilon)(A_{qi} + B_{1q_i}K_i)^T P_i(A_{qi} + B_{1q_i}K_i) - (1 + \frac{1}{\epsilon})(\Delta(A_{qi} + B_{1q_i}K_i))^T P_i \\
& \times \Delta(A_{qi} + B_{1q_i}K_i), \\
\psi_{24} = {} & D_{2i}\{B_{2q_i}^T P_i B_{2q_i} + \Delta B_{2q_i}^T P_i \Delta B_{2q_i} + H_{1q_i} P_i H_{1q_i}^T + \Delta H_{1q_i}^T P_i \Delta H_{1q_i} + 3(\epsilon B_{2q_i} P_i \\
& \times B_{2q_i}^T + \frac{1}{\epsilon}\Delta B_{2q_i}^T P_i \Delta B_{2q_i}) + 2\epsilon H_{1q_i} P_i H_{1q_i}^T + \frac{1}{\epsilon} H_{1q_i} P_i H_{1q_i}^T + \epsilon \Delta B_{2q_i}^T P_i \Delta B_{2q_i} \\
& + \frac{2}{\epsilon}\Delta H_{1q_i}^T P_i \Delta H_{1q_i}\} + D_{2i}^{-1} P_i - I \\
\psi_{31} = {} & (1+\epsilon)(A_{h_i} + B_{1h_i}K_i)^T P_i(A_{h_i} + B_{1h_i}K_i) + (1 + \frac{1}{\epsilon})(\Delta(A_{h_i} + B_{1h_i}K_i))^T P_i \\
& \times \Delta(A_{h_i} + B_{1h_i}K_i), \\
\psi_{33} = {} & -\frac{1}{2}(C_{hi} + C_{hi}^{-1})R, \\
\psi_{44} = {} & I, \\
\psi_{15} = {} & D_{2i}\{B_{2i}^T P_i B_{2i} + \Delta B_{2i}^T P_i \Delta B_{2i} + H_{1i} P_i H_{1i}^T + \Delta H_{1i}^T P_i \Delta H_{1i} + 3(\epsilon B_{2i} P_i B_{2i}^T \\
& + \frac{1}{\epsilon}\Delta B_{2i}^T P_i \Delta B_{2i}) + 2\epsilon H_{1i} P_i H_{1i}^T + \frac{1}{\epsilon} H_{1i} P_i H_{1i}^T + \epsilon \Delta B_{2i}^T P_i \Delta B_{2i} + \frac{2}{\epsilon}\Delta H_{1i}^T P_i \Delta H_{1i}\} + D_{2i}^{-1} P_i, \\
& \Omega_{55} = -\gamma^2 I.
\end{aligned}$$

If

$$\int_0^t E[z^T(s)z(s)]\mathrm{ds} \le \gamma^2 \int_0^{\mathrm{t}} [\mathrm{w}^{\mathrm{T}}(\mathrm{s})\mathrm{w}(\mathrm{s})]\mathrm{ds},$$

and subsequently that $E\|z(t)\| \le \gamma \|w(t)\|_{L^2}$, therefore closed-loop system (1–3) is robust stochastically stable. If Theorem 4.1 can be applied in the form of Schur complement, then $J(t) < 0, \Gamma_i < 0$, for all $t > 0$. Let $P_i = X_i^{-1}$, $Y_i = K_i X_i$, $T_i = \mathrm{diag}(X_i, I, I, I, I)$, then multiplying (19) by T_i and $\mathrm{diag}(P_i^{-1}, I, I, I, I)$, we can find the coupled matrix inequalities as follows

$$\begin{pmatrix} \chi_{1i} & \Theta_i & \Psi_{1i} & 0 & \Psi_{2i} \\ \Theta_i^T & -\Phi_{1i} & 0 & \Phi_{2i}^T & 0 \\ \Psi_{1i}^T & 0 & -\mu_i & 0 & 0 \\ 0 & \Phi_{2i}^T & 0 & -I & 0 \\ \Psi_{2i}^T & 0 & 0 & 0 & -\gamma^2 I \end{pmatrix} < 0, \tag{20}$$

where

$$
\begin{aligned}
\chi_{1i} &= \begin{pmatrix} \chi_{2i} & \Pi_i^T \\ \Pi_i & -\frac{1}{2}(C_{hi}+C_{hi}^{-1})R \end{pmatrix} > 0, \\
\chi_{2i} &= C_i\{(A_iX_i^T + B_{1i}Y_i^T)(A_iX_i + B_{1i}Y_i) + \Delta(A_iX_i^T + B_{1i}Y_i^T)\Delta(A_iX_i + B_{1i}Y_i) \\
&\quad + E_iX_iE_i^T + \Delta E_i^T X_i \Delta E_i + 3\epsilon(A_iX_i + B_{1i}Y_i)(A_iX_i^T + B_{1i}Y_i^T) + \frac{1}{\epsilon}\Delta(A_iX_i^T \\
&\quad + B_{1i}Y_i^T)\Delta(A_iX_i + B_{1i}Y_i) + 2\epsilon E_iX_iE_i^T + \frac{1}{\epsilon}E_iX_iE_i^T + \epsilon(\Delta(A_iX_i^T + B_{1i}Y_i^T)) \\
&\quad \times \Delta(A_iX_i + B_{1i}Y_i) + \frac{2}{\epsilon}\Delta E_i^T X_i \Delta E_i\} + X_i^{-1}C_i^{-1}X_i + X_i(Q+R)^T X_i^{-1} \\
&\quad + X_i(Q+R)X_i^{-1} + \Lambda_{ii}X_i, \\
\Pi_i &= ((1+\epsilon)(A_{h_i} + B_{1h_i}K_i)^T P_i (A_{h_i} + B_{1h_i}K_i) + (1+\frac{1}{\epsilon})(\Delta(A_{h_i} + B_{1h_i}K_i))^T \\
&\quad \times P_i \Delta(A_{h_i} + B_{1h_i}K_i))^T, \\
\Theta_i &= 2(C_i\{(A_{qi} + B_{1q_i}K_i)^T P_i (A_{qi} + B_{1q_i}K_i) + (\Delta(A_{qi} + B_{1q_i}K_i))^T P_i \Delta(A_{qi} + B_{1q_i}K_i) \\
&\quad + E_{qi}P_iE_{qi}^T + \Delta E_{qi}^T P_i \Delta E_{qi} + 3(\epsilon(A_{qi} + B_{1q_i}K_i)^T P_i (A_{qi} + B_{1q_i}K_i) \\
&\quad + \frac{1}{\epsilon}(\Delta(A_{qi} + B_{1q_i}K_i))^T P_i \Delta(A_{qi} + B_{1q_i}K_i) + 2\epsilon E_{qi}P_iE_{qi}^T + \frac{1}{\epsilon}E_{qi}P_iE_{qi}^T \\
&\quad + \epsilon(\Delta(A_{qi} + B_{1q_i}K_i))^T P_i \Delta(A_{qi} + B_{1q_i}K_i) + \frac{2}{\epsilon}\Delta E_{qi}^T P_i \Delta E_{qi}\} + C_i^{-1}P_i \\
&\quad + P_i(Q+R)^T + P_i(Q+R)), \\
\Phi_{1i} &= C_i\{(A_{qi} + B_{1q_i}K_i)^T P_i (A_{qi} + B_{1q_i}K_i) + (\Delta(A_{qi} + B_{1q_i}K_i))^T P_i \Delta(A_{qi} + B_{1q_i}K_i) \\
&\quad + E_{qi}P_iE_{qi}^T + \Delta E_{qi}^T P_i \Delta E_{qi} + 3(\epsilon(A_{qi} + B_{1q_i}K_i)^T P_i (A_{qi} + B_{1q_i}K_i) + \frac{1}{\epsilon}(\Delta(A_{qi} \\
&\quad + B_{1q_i}K_i))^T P_i \Delta(A_{qi} + B_{1q_i}K_i)) + 2\epsilon E_{qi} \times P_iE_{qi}^T + \frac{1}{\epsilon}E_{qi}P_iE_{qi}^T + \epsilon(\Delta(A_{qi} + B_{1q_i}K_i))^T \\
&\quad \times P_i \Delta(A_{qi} + B_{1q_i}K_i) + \frac{2}{\epsilon}\Delta E_{qi}^T P_i \Delta E_{qi}\} + C_i^{-1}P_i + P_i(Q+R)^T + P_i(Q+R) - W \\
\Psi_{1i} &= \sqrt{\Lambda_{i1}}X_i, \sqrt{\Lambda_{i2}}X_i, \cdots, \sqrt{\Lambda_{iN}}X_i, \\
\mu_i &= \mathrm{diag}(\mathrm{X_1}, \mathrm{X_2}, \ldots, \mathrm{X_{i-1}}, \mathrm{X_i}, \mathrm{X_{i+1}}, \mathrm{X_N}), \\
\Phi_{2i} &= D_{2i}\{B_{2q_i}^T P_i B_{2q_i} + \Delta B_{2q_i}^T P_i \Delta B_{2q_i} + H_{1q_i}P_iH_{1q_i}^T + \Delta H_{1q_i}^T P_i \Delta H_{1q_i} + 3(\epsilon B_{2q_i} \\
&\qquad \times P_i B_{2q_i}^T + \frac{1}{\epsilon}\Delta B_{2q_i}^T P_i \Delta B_{2q_i}) + 2\epsilon H_{1q_i}P_iH_{1q_i}^T + \frac{1}{\epsilon}H_{1q_i}P_iH_{1q_i}^T \\
&\quad + \epsilon \Delta B_{2q_i}^T P_i \Delta B_{2q_i} + \frac{2}{\epsilon}\Delta H_{1q_i}^T P_i \Delta H_{1q_i}\} + D_{2i}^{-1}P_i - I \\
\Psi_{2i} &= D_{2i}\{B_{2i}^T P_i B_{2i} + \Delta B_{2i}^T P_i \Delta B_{2i} + H_{1i}P_iH_{1i}^T + \Delta H_{1i}^T P_i \Delta H_{1i} + 3(\epsilon B_{2i} \\
&\quad \times P_i B_{2i}^T + \frac{1}{\epsilon}\Delta B_{2i}^T P_i \Delta B_{2i}) + 2\epsilon H_{1i}P_iH_{1i}^T + \frac{1}{\epsilon}H_{1i}P_iH_{1i}^T \\
&\quad + \epsilon \Delta B_{2i}^T P_i \Delta B_{2i} + \frac{2}{\epsilon}\Delta H_{1i}^T P_i \Delta H_{1i}\} + D_{2i}^{-1}P_i.
\end{aligned}
$$

Hence, system (1)–(3) is roust stochastically stable with γ attenuation level. □

Numerical examples

In this section, we give some numerical examples to demonstrate the effectiveness of the proposed results.

Example 1 Consider the neutral system (1)–(4) with $K = Y_i X_i^{-1}$, and assume that following uncertainty matrix holds, the system (1) is stable if we choose state $x(t)$ in R^3 and Markov chain $r(t)$ in $S = \{1, 2, 3\}$.

$$
\begin{aligned}
\Delta A_i &= \begin{bmatrix} 0.7 & -0.2 \\ 0.7 & 0.1 \end{bmatrix}, \Delta B_{1i} = \begin{bmatrix} 0.7 & -0.2 \\ 0.3 & -0.1 \end{bmatrix}, \\
\Delta E_i &= \begin{bmatrix} 0 & 1 \\ 0.1 & 0.4 \end{bmatrix}, \Delta A_{qi} = \begin{bmatrix} -0.2 & 0.5 \\ 1 & -0.3 \end{bmatrix}, \\
\Delta E_{qi} &= \begin{bmatrix} 0 & 1 \\ -0.2 & 0.4 \end{bmatrix}, Q = \begin{bmatrix} 0.5 & -0.2 \\ 0.1 & -0.1 \end{bmatrix}, \\
R &= \begin{bmatrix} -0.1 & 0.2 \\ -0.1 & 0.3 \end{bmatrix}, \Delta A_{hi} = \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.1 \end{bmatrix}, \\
W &= \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, E_{hi} = \begin{bmatrix} 0.1 & 0.3 \\ 0.2 & 0.3 \end{bmatrix}, \\
B_{1hi} &= \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}, B_{1qi} = \begin{bmatrix} 1 & 0.1 \\ -0.2 & 0.3 \end{bmatrix}, \\
\Lambda_{ij} &= \begin{bmatrix} -0.2 & 0.5 \\ 0.2 & 0.5 \end{bmatrix}, C_{h_i} = 0.1.
\end{aligned}
$$

By Theorem 3.2, the closed-loop system (1–4) is stochastically mean square stable with respect of above uncertainty. The stability trajectory of (1)–(4) is shown in Fig. 1 (when $\epsilon = 1$). This example shows that the value of $x_1 = x(t)$, $x_2 = x(t-q)$ and $x_3 = x(t-h)$ and we point that proposed algorithm result is more convenient in delayed sampled data and stochastic parameter systems. The responses of state feedback gain are

$$
\begin{aligned}
K_i &= \begin{bmatrix} 77.2972 & -205.6894 \\ 206.8689 & 121.4651 \end{bmatrix}, \\
X_i &= \begin{bmatrix} 19.2423 & -22.8123 \\ -22.8123 & 12.2453 \end{bmatrix}, \\
Y_i &= \begin{bmatrix} 6.3124 & -9.0166 \\ 9.0683 & 6.3124 \end{bmatrix}.
\end{aligned}
$$

This objective is developed in MATLAB-LMI Control Toolbox.

Example 2 Consider the neutral system (1)–(4) with $K = Y_i X_i^{-1}$, and assume that following uncertainty matrix holds, the system (1)–(4) is roust stochastically stable with γ attenuation level if we choose state $x(t)$ in R^3 and Markov chain $r(t)$ in $S = \{1, 2, 3\}$.

$$\Delta A_i = \begin{bmatrix} 0.7 & -0.2 \\ 0.7 & 0.1 \end{bmatrix}, \Delta B_{1i} = \begin{bmatrix} 0.7 & -0.2 \\ 0.3 & -0.1 \end{bmatrix},$$
$$\Delta E_i = \begin{bmatrix} 0 & 1 \\ 0.1 & 0.4 \end{bmatrix}, \Delta A_{qi} = \begin{bmatrix} -0.2 & 0.5 \\ 1 & -0.3 \end{bmatrix},$$
$$\Delta E_{qi} = \begin{bmatrix} 0 & 1 \\ -0.2 & 0.4 \end{bmatrix}, Q = \begin{bmatrix} 0.5 & -0.2 \\ 0.1 & -0.1 \end{bmatrix},$$
$$R = \begin{bmatrix} -0.1 & 0.2 \\ -0.1 & 0.3 \end{bmatrix}, \Delta A_{hi} = \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.1 \end{bmatrix},$$
$$W = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, E_{hi} = \begin{bmatrix} 0.1 & 0.3 \\ 0.2 & 0.3 \end{bmatrix},$$
$$B_{1hi} = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}, B_{1qi} = \begin{bmatrix} 1 & 0.1 \\ -0.2 & 0.3 \end{bmatrix},$$
$$\Lambda_{ij} = \begin{bmatrix} -0.2 & 0.5 \\ 0.2 & 0.5 \end{bmatrix}, C_{h_i} = 0.1,$$
$$D_{2i} = 0.2, \gamma = 0.0517.$$

By Theorem 4.1, the closed-loop system (1)–(4) is roust stochastically stable with γ attenuation level with respect of above uncertainty. The stability trajectory of (1)–(4) is shown in Fig. 2 (when $\epsilon = 0.25$) and system responses feedback gain is showed in below and the convergence level of control design is showed in Table 1. This example shows that the value of $x_1 = x(t)$, $x_2 = x(t - q)$ and $x_3 = x(t - h)$ and we point that proposed algorithm result is more convenient in delayed sampled data and comparison is made some of the references at convergence level.

$$K_i = \begin{bmatrix} 135.0374 & -232.7725 \\ 234.1072 & 244.2918 \end{bmatrix},$$
$$X_i = \begin{bmatrix} 29.3426 & -25.8160 \\ -25.8160 & 16.2453 \end{bmatrix},$$
$$Y_i = \begin{bmatrix} 8.3124 & -9.0166 \\ 9.0683 & 8.3255 \end{bmatrix}.$$

Remark 1 In example 1 if D_{2i} value is $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $\gamma = 1$ and in example 2 if D_{2i} value is $\begin{bmatrix} 0.2 & 0 \\ 0 & 0 \end{bmatrix}$ and $\gamma = 0.0517$, then comparing these two example of K_i, X_i and Y_i, values, only example 2 gives less conservative. If we choose more than (or) less than of 0.2 in D_{2i}, the system trajectory is not vanishing. And also comparing reference [2, 22, 23], the existing result is more accurate. This example shows that our method could be lower attenuation level than existing result [2, 22, 23].

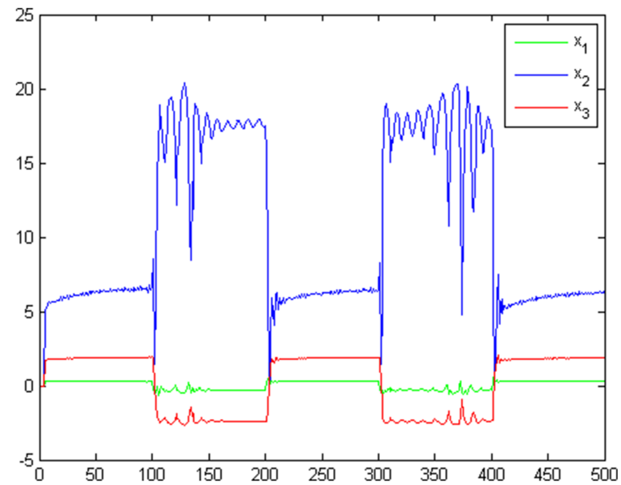

Fig. 1 The behavior of state response $x_1(t) = x(t), x_2(t) = x(t-h), x_3(t) = x(t-q)$

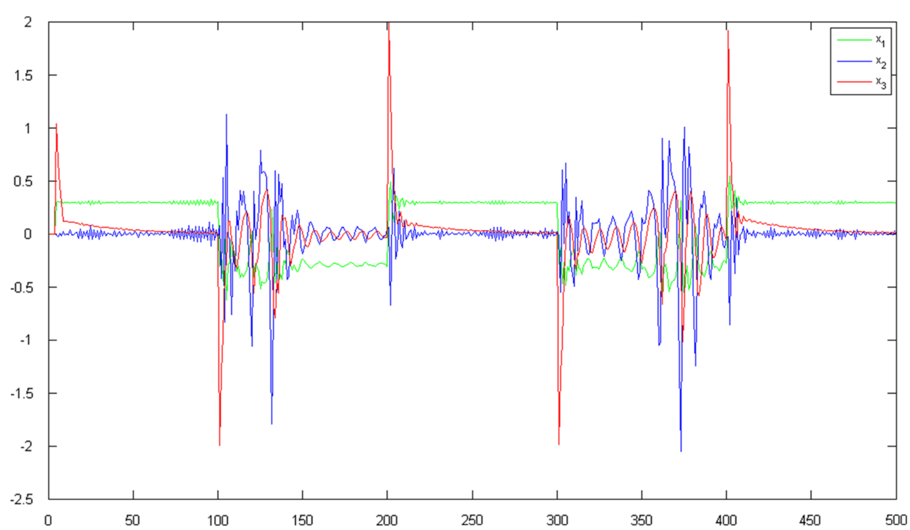

Fig. 2 The disturbance $w(t)$, when $\epsilon = 0.25$.

Table 1 The convergence level at $\epsilon = 0.5$

Ref. no	Convergence level	ϵ value
[2]	3.38039	0.5
[9]	3.06891	0.5
[17]	3.02053	0.5
[22]	4.21003	0.5
[23]	3.94212	0.5
Proposed algorithm result	2.18320	0.5

Conclusions

In this article, we looked into the Markov chain-based design of stochastic neural state delay systems. It offers predetermined uncertainties and stochastic neutral system's discrete state space delay is stochastically mean square stable. In the H_∞ control design, the output feedback is used, whereas the LMIS filters design was employed to demonstrate robust stochastic stability. The design challenges were to turn a stochastic system's state space delay into neutrality and solve a few reduced-order errors that converge to zero. Compared with the previous works, the main results of this paper have several features: (i) time delays can exist in control input and the measurement output, (ii) The uncertainty can appear in all system matrices. The proposed systems with the H_∞ control method can be achieved a lower attenuation level using the (iii) delay differential method [2]. The future research will be development of stabilization of some fractional delay systems of neutral type. The mathematical examples have shown significant improvements over some existing results.

Author contributions

R.R.K.: Conceptualization, Methodology, R.N.: Validation, Writing—original draft, V.A.: Writing—review & editing, A.S.: simulation development. All authors read and approved the final manuscript.

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Data availibility

Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

Declarations

Competing interests

The author declares that there is no competing interests in this paper.

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