Pankratov
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A model for prognosis of influence of radiation dose on value of charge carrier mobility: an analytical approach for analysis of the introduced model

Evgeny L. Pankratov[1,2*]

*Correspondence: elp2004@mail.ru

[1] Nizhny Novgorod State University, 23 Gagarin Avenue, Nizhny Novgorod, Russia 603950
[2] Nizhny Novgorod State Technical University, 24 Minin Street, Nizhny Novgorod, Russia 603950

Abstract

In this paper, we analyzed the dependence of charge carriers mobility on value of radiation dose during ion implantation. Based on the considered model, we determine conditions to decrease radiation damage in the irradiated materials. Also we introduce an analytical approach to analyze mass transfer in the irradiated materials. The approach gives a possibility to take into account nonlinearity of the considered process, as well as changes of parameters of the considered process in space and time at one time.

Keywords: Radiation damage, Charge carriers mobility, Influence of value of radiation dose

Introduction

Currently, an actual point of solid-state electronic is elaboration of new electronic devices and improvement of characteristics of previously developed ones [1–5]. To solve these problems, both (i) the technological processes, which were used to manufacture the considered devices, and (ii) the characteristics of already manufactured devices are attracted an interest. In this paper, we consider a multilayer structure, which consists of a substrate and an epitaxial layer (see Fig. 1). We assume that a dopant was implanted into the epitaxial layer to generate the required type of conductivity (n or p) in the doped are (for example, during manufacturing of a p–n junction). Main aim of this paper is introduction of model for analysis of dependence of charge carriers mobility on radiation dose in comparison with models, which were considered in literature. Based on the model we analyzed mass transport in the considered multilayer structure in more details to increase of radiation resistance of solid-state electronic devices. An accompanying aim of this work is development of an analytical approach for analysis of mass transfer, which simultaneously takes into account of nonlinearity of the considered process, as well as changes of the considered parameters in space and time.

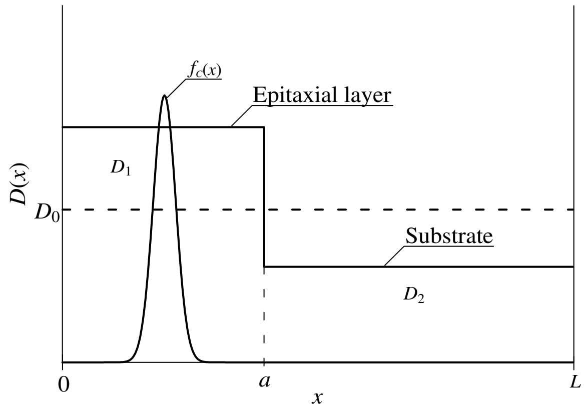

Fig. 1 Two-layer structure, which consist of a substrate and an epitaxial layer

Method of solution

To solve our aim, we calculate and analyzed the spatiotemporal distribution of concentration of the considered dopant in the above multilayer structure. We calculate the above distribution of dopant by solving the second Fick's law in the following form [6–8]

$$\frac{\partial C(x,t)}{\partial t} = \frac{\partial}{\partial x}\left[D_C \frac{\partial C(x,t)}{\partial x}\right] \quad (1)$$

Boundary and initial conditions for the above equation could be written as

$$\left.\frac{\partial C(x,t)}{\partial x}\right|_{x=0} = 0, \ \left.\frac{\partial C(x,t)}{\partial x}\right|_{x=L} = 0, \ C(x,0) = f_C(x).$$

Here $C(x, t)$ is the spatiotemporal distribution of concentration of dopant; D is the dopant diffusion coefficient. Values of dopant diffusion coefficient depend on properties of materials of multilayer structure, speed of heating and cooling of materials during annealing and spatiotemporal distribution of concentration of dopant. Dependences of dopant diffusions coefficients on parameters could be approximated by the following relation [6–8]

$$D_C = D_L(x,T)\left[1 + \xi \frac{C^\gamma(x,t)}{P^\gamma(x,T)}\right]\left[1 + \varsigma_1 \frac{V(x,t)}{V^*} + \varsigma_2 \frac{V^2(x,t)}{(V^*)^2}\right] \quad (2)$$

Here D_L (x,T) is the spatial (due to accounting all layers of multilayer structure) and temperature (due to Arrhenius law) dependences of dopant diffusion coefficient; T is the temperature of annealing; P (x,T) is the limit of solubility of dopant; parameter γ depends on properties of materials and could be integer usially in the following interval $\gamma \in [1, 3]$ [6]; and V (x,t) is the spatiotemporal distribution of concentration of radiation vacancies with the equilibrium distribution V^*. Concentrational dependence of dopant diffusion coefficient has been described in detail in [6]. Spatiotemporal distributions of concentration of point radiation defects have been determined by solving the following system of equations [6–8]

$$\begin{cases} \dfrac{\partial I(x,t)}{\partial t} = \dfrac{\partial}{\partial x}\left[D_I(x,T)\dfrac{\partial I(x,t)}{\partial x}\right] - k_{I,I}(x,T)\,I^2(x,t) \\ \qquad - k_{I,V}(x,T)\,I(x,t)\;V(x,t) \\ \dfrac{\partial V(x,t)}{\partial t} = \dfrac{\partial}{\partial x}\left[D_V(x,T)\dfrac{\partial V(x,t)}{\partial x}\right] - k_{V,V}(x,T)\,V^2(x,t) \\ \qquad - k_{I,V}(x,T)\,I(x,t)\;V(x,t) \end{cases} \tag{3}$$

Boundary and initial conditions for the above equations could be written as

$$\left.\frac{\partial I(x,t)}{\partial x}\right|_{x=0} = 0,\ \left.\frac{\partial I(x,t)}{\partial x}\right|_{x=L} = 0,\ \left.\frac{\partial V(x,t)}{\partial x}\right|_{x=0} = 0,\ \left.\frac{\partial V(x,t)}{\partial x}\right|_{x=L} = 0, \\ I(x,0) = f_I(x),\, V(x,0) = f_V(x). \tag{4}$$

Here $I(x,t)$ is the spatiotemporal distribution of concentration of radiation interstitials with the equilibrium distribution I^*; $D_I(x,T)$ and $D_V(x,T)$ are the diffusion coefficients of interstitials and vacancies, respectively; terms $V^2(x,t)$ and $I^2(x,t)$ decribe generation of divacancies and diinterstitials, respectively (see, for example, [8] and appropriate references in this book); and $k_{I,V}(x,T)$, $k_{I,I}(x,T)$ and $k_{V,V}(x,T)$ are the parameters of recombination of point radiation defects and generation of their complexes. Spatiotemporal distributions of divacancies $\Phi_V(x,t)$ and diinterstitials $\Phi_I(x,t)$ could be determined by solving the following system of equations [7, 8]

$$\begin{cases} \dfrac{\partial \Phi_I(x,t)}{\partial t} = \dfrac{\partial}{\partial x}\left[D_{\Phi_I}(x,T)\dfrac{\partial \Phi_I(x,t)}{\partial x}\right] + k_I(x,T)\,I(x,t) \\ \qquad + k_{I,I}(x,T)\,I^2(x,t) \\ \dfrac{\partial \Phi_V(x,t)}{\partial t} = \dfrac{\partial}{\partial x}\left[D_{\Phi_V}(x,T)\dfrac{\partial \Phi_V(x,t)}{\partial x}\right] + k_V(x,T)\,V(x,t) \\ \qquad + k_{V,V}(x,T)\,V^2(x,t) \end{cases} \tag{5}$$

Boundary and initial conditions for the above equations could be written as

$$\left.\frac{\partial \Phi_I(x,t)}{\partial x}\right|_{x=0} = 0,\ \left.\frac{\partial \Phi_I(x,t)}{\partial x}\right|_{x=L} = 0,\ \left.\frac{\partial \Phi_V(x,t)}{\partial x}\right|_{x=0} = 0,\ \left.\frac{\partial \Phi_V(x,t)}{\partial x}\right|_{x=L} = 0, \\ \Phi_I(x,0) = f_{\Phi I}(x),\ \Phi_V(x,0) = f_{\Phi V}(x). \tag{6}$$

Here $D_{\Phi I}(x,T)$ and $D_{\Phi V}(x,T)$ are the diffusion coefficients of simplest complexes of radiation defects; $k_I(x,T)$ and $k_V(x,T)$ are the parameters of decay of these complexes of radiation defects.

We calculate spatiotemporal distributions of the considered concentrations of dopant and radiation defects by solving Eqs. (1), (3) and (5) in the framework of method of averaging of function corrections [9–11]. Previously let us transform Eqs. (1), (3) and (5) to the following form with account initial distributions of the considered concentrations

$$\frac{\partial C(x,t)}{\partial t} = \frac{\partial}{\partial x}\left[D_C\frac{\partial C(x,t)}{\partial x}\right] + f_C(x)\,\delta(t) \tag{1a}$$

$$\begin{cases} \dfrac{\partial I(x,t)}{\partial t} = \dfrac{\partial}{\partial x}\left[D_I(x,T)\dfrac{\partial I(x,t)}{\partial x}\right] - k_{I,I}(x,T)\,I^2(x,t) \\ \qquad - k_{I,V}(x,T)\;I(x,t)\;V(x,t) + f_I(x)\,\delta(t) \\ \dfrac{\partial V(x,t)}{\partial t} = \dfrac{\partial}{\partial x}\left[D_V(x,T)\dfrac{\partial V(x,t)}{\partial x}\right] - k_{V,V}(x,T)\,V^2(x,t) \\ \qquad - k_{I,V}(x,T)\,I(x,t)\;V(x,t) + f_V(x)\,\delta(t) \end{cases} \tag{3a}$$

$$\begin{cases} \dfrac{\partial \Phi_I(x,t)}{\partial t} = \dfrac{\partial}{\partial x}\left[D_{\Phi_I}(x,T)\dfrac{\partial \Phi_I(x,t)}{\partial x}\right] + k_I(x,T)\,I(x,t) \\ \qquad + k_{I,I}(x,T)\,I^2(x,t) + f_{\Phi_I}(x)\,\delta(t) \\ \dfrac{\partial \Phi_V(x,t)}{\partial t} = \dfrac{\partial}{\partial x}\left[D_{\Phi_V}(x,T)\dfrac{\partial \Phi_V(x,t)}{\partial x}\right] + k_V(x,T)\,V(x,t) \\ \qquad + k_{V,V}(x,T)\,V^2(x,t) + f_{\Phi_V}(x)\,\delta(t) \end{cases} \tag{5a}$$

Now in the framework of the method let us replace the concentrations of dopant and radiation defects in the right sides of Eqs. (1a), (3a) and (5a) on their not yet known average values $\alpha_{1\rho}$. In this situation, we obtain equations to calculate the first-order approximations of the required concentrations in the following form

$$\frac{\partial C_1(x,t)}{\partial t} = f_C(x)\,\delta(t) \tag{1b}$$

$$\begin{cases} \dfrac{\partial I_1(x,t)}{\partial t} = f_I(x)\,\delta(t) - \alpha_{1I}^2 k_{I,I}(x,T) - \alpha_{1I}\alpha_{1V} k_{I,V}(x,T) \\ \dfrac{\partial V_1(x,t)}{\partial t} = f_V(x)\,\delta(t) - \alpha_{1V}^2 k_{V,V}(x,T) - \alpha_{1I}\alpha_{1V} k_{I,V}(x,T) \end{cases} \tag{3b}$$

$$\begin{cases} \dfrac{\partial \Phi_{1I}(x,t)}{\partial t} = f_{\Phi_I}(x)\,\delta(t) + k_I(x,T)\,I(x,t) + k_{I,I}(x,T)\,I^2(x,t) \\ \dfrac{\partial \Phi_{1V}(x,t)}{\partial t} = f_{\Phi_V}(x)\,\delta(t) + k_V(x,T)\,I(x,t) + k_{V,V}(x,T)\,V^2(x,t) \end{cases} \tag{5b}$$

Integration of the left and right sides of the above Eqs. (1b), (3b) and (5b) on time gives us possibility to obtain relations for above approximation in the final form

$$C_1(x,t) = f_C(x) \tag{1c}$$

$$\begin{cases} I_1(x,t) = f_I(x) - \alpha_{1I}^2 \displaystyle\int_0^t k_{I,I}(x,T)\,\mathrm{d}\tau - \alpha_{1I}\alpha_{1V} \int_0^t k_{I,V}(x,T)\,\mathrm{d}\tau \\ V_1(x,t) = f_V(x) - \alpha_{1V}^2 \displaystyle\int_0^t k_{V,V}(x,T)\,\mathrm{d}\tau - \alpha_{1I}\alpha_{1V} \int_0^t k_{I,V}(x,T)\,\mathrm{d}\tau \end{cases} \tag{3c}$$

$$\begin{cases} \Phi_{1I}(x,t) = f_{\Phi_I}(x) + \int\limits_0^t k_I(x,T)\, I(x,\tau)\, \mathrm{d}\tau + \int\limits_0^t k_{I,I}(x,T)\, I^2(x,\tau)\, \mathrm{d}\tau \\ \Phi_{1V}(x,t) = f_{\Phi_V}(x) + \int\limits_0^t k_V(x,T)\, V(x,\tau)\, \mathrm{d}\tau + \int\limits_0^t k_{V,V}(x,T)\, V^2(x,\tau)\, \mathrm{d}\tau \end{cases} \tag{5c}$$

We calculate average values of the first-order approximations of concentrations of dopant and radiation defects by using the following standard relation [9–11]

$$\alpha_{1\rho} = \frac{1}{\Theta L} \int\limits_0^{\Theta} \int\limits_0^{L} \rho_1(x,t)\, \mathrm{d}x\, \mathrm{d}t \tag{7}$$

Substitution of the relations (1c), (3c) and (5c) in the relation (7) gives us the possibility to obtain required average values in the following form

$$\alpha_{1C} = \frac{1}{L} \int\limits_0^L f_C(x)\, \mathrm{d}x, \quad \alpha_{1I} = \sqrt{\frac{(a_3 + A)^2}{4\, a_4^2} - 4\left(B + \frac{\Theta\, a_3 B + \Theta^2 L\, a_1}{a_4}\right)} - \frac{a_3 + A}{4\, a_4},$$

$$\alpha_{1V} = \frac{1}{S_{IV\,00}} \left[\frac{\Theta}{\alpha_{1I}} \int\limits_0^{L_x} f_I(x)\, \mathrm{d}x - \alpha_{1I} S_{II\,00} - \Theta L \right]$$

where $S_{\rho\rho\; ij} = \int\limits_0^{\Theta} (\Theta - t) \int\limits_0^{L} k_{\rho,\rho}(x,T)\, I_1^i(x,t)\, V_1^j(x,t)\, \mathrm{d}x\, \mathrm{d}t$, $a_4 = S_{II\,00}\left(S_{IV\,00}^2 - S_{II\,00} S_{VV\,00}\right)$, $a_3 = S_{IV00} S_{II00} + S_{IV00}^2 - S_{II00} S_{VV00}$, $a_2 = S_{IV00} S_{IV00}^2 \int\limits_0^L f_V(x)\, \mathrm{d}x + 2\, S_{VV00} S_{II00} \int\limits_0^L f_I(x)\, \mathrm{d}x$ $+ S_{IV00} \Theta L^2 - \Theta L^2 - S_{IV00}^2 \int\limits_0^L f_I(x)\, \mathrm{d}x$, $a_1 = S_{IV00} \int\limits_0^L f_I(x)\, \mathrm{d}x$, $a_0 = S_{VV00} \left[\int\limits_0^L f_I(x)\, \mathrm{d}x \right]^2$, $A = \sqrt{8\, y + \Theta^2 \frac{a_3^2}{a_4^2} - 4\, \Theta \frac{a_2}{a_4}}$, $B = \frac{\Theta\, a_2}{6\, a_4} + \sqrt[3]{\sqrt{q^2 + p^3} - q} - \sqrt[3]{\sqrt{q^2 + p^3} + q}$, $q = \frac{\Theta^3 a_2}{24\, a_4^2}$ $\times \left(4 a_0 - \Theta L \frac{a_1 a_3}{a_4}\right) - \Theta^3 \frac{a_0}{8 a_4^2} \left(4\, a_2 - \Theta \frac{a_3^2}{a_4}\right) - \frac{\Theta^3 a_2^3}{54\, a_4^3} - L^2 \frac{\Theta^4 a_1^2}{8\, a_4^2}$, $p = \Theta^2 \frac{4 a_0 a_4 - \Theta L\, a_1 a_3}{12\, a_4^2}$ $- \Theta\, a_2 / 18\, a_4$,

$$\alpha_{1\Phi_I} = \frac{R_{I1}}{\Theta L} + \frac{S_{II\,20}}{\Theta L} + \frac{1}{L} \int\limits_0^L f_{\Phi_I}(x)\, \mathrm{d}x, \quad \alpha_{1\Phi_V}$$

$$= \frac{R_{V1}}{\Theta L} + \frac{S_{VV\,20}}{\Theta L} + \frac{1}{L} \int\limits_0^L f_{\Phi_V}(x)\, \mathrm{d}x,$$

where $R_{\rho\; i} = \int\limits_0^{\Theta} (\Theta - t) \int\limits_0^{L} k_I(x,T)\, I_1^i(x,t)\, \mathrm{d}x\, \mathrm{d}t$.

Now let us calculate approximations of the second and the higher orders of the considered concentrations of dopant and radiation defects in the framework of the standard iterative procedure of the method of averaging of function corrections [9–11]. In the framework of the procedure to calculate approximations of the n-th order of concentrations of dopant and radiation defects, we replace the required concentrations in

Eqs. (1c), (3c), (5c) on the following sum $\alpha_{n\rho}+\rho_{\mathrm{n}-1}$ (x,t). The replacement leads to the following transformation of the appropriate equations

$$\frac{\partial\, C_2(x,t)}{\partial\, t}=\frac{\partial}{\partial\, x}\left(D_L(x,T)\left\{1+\xi\frac{[\alpha_{2C}+C_1(x,t)]^{\gamma}}{P^{\gamma}(x,T)}\right\}\left[1+\varsigma_1\frac{V\,(x,t)}{V^*}+\varsigma_2\frac{V^2(x,t)}{(V^*)^2}\right]\frac{\partial\, C_1(x,t)}{\partial\, x}\right)+f_C\,(x)\;\delta(t) \tag{1d}$$

$$\begin{cases}\dfrac{\partial I_2(x,t)}{\partial t}=\dfrac{\partial}{\partial x}\left[D_I(x,T)\dfrac{\partial I_1(x,t)}{\partial x}\right]-k_{I,I}(x,T)\,[\alpha_{1I}+I_1(x,t)]^2\\ \qquad -k_{I,V}(x,T)\,[\alpha_{1I}+I_1(x,t)]\,[\alpha_{1V}+V_1(x,t)]\\ \dfrac{\partial V_2(x,t)}{\partial t}=\dfrac{\partial}{\partial x}\left[D_V(x,T)\dfrac{\partial V_1(x,t)}{\partial x}\right]-k_{V,V}(x,T)\,[\alpha_{1V}+V_1(x,t)]^2\\ \qquad -k_{I,V}(x,T)\,[\alpha_{1I}+I_1(x,t)]\,[\alpha_{1V}+V_1(x,t)]\end{cases} \tag{3d}$$

$$\begin{cases}\dfrac{\partial \Phi_{2I}(x,t)}{\partial t}=\dfrac{\partial}{\partial x}\left[D_{\Phi_I}(x,T)\dfrac{\partial \Phi_{1I}(x,t)}{\partial x}\right]+k_{I,I}(x,T)\,I^2(x,t)\\ \qquad +k_I(x,T)\,I\,(x,t)+f_{\Phi_I}\,(x)\,\delta(t)\\ \dfrac{\partial \Phi_{2V}(x,t)}{\partial t}=\dfrac{\partial}{\partial x}\left[D_{\Phi_V}(x,T)\dfrac{\partial \Phi_{1V}(x,t)}{\partial x}\right]+k_{V,V}(x,T)\,V^2(x,t)\\ \qquad +k_V(x,T)\,V\,(x,t)+f_{\Phi_V}\,(x)\,\delta(t)\end{cases} \tag{5d}$$

Integration of the left and the right sides of the above Eqs. (1d), (3d) and (5d) gives us possibility to obtain relations for the required concentrations in the final form

$$C_2(x,t)=f_C\,(x)+\frac{\partial}{\partial x}\int\limits_0^t D_L(x,T)\,\left\{1+\xi\frac{[\alpha_{2C}+C_1(x,\tau)]^{\gamma}}{P^{\gamma}(x,T)}\right\}\left[1+\varsigma_1\frac{V\,(x,\tau)}{V^*}+\varsigma_2\frac{V^2(x,\tau)}{(V^*)^2}\right]\,\frac{\partial C_1(x,\tau)}{\partial x}\,\mathrm{d}\tau \tag{1e}$$

$$\begin{cases}I_2(x,t)=\dfrac{\partial}{\partial x}\displaystyle\int\limits_0^t D_I(x,T)\dfrac{\partial I_1(x,\tau)}{\partial x}\mathrm{d}\,\tau+f_I(x)-\int\limits_0^t k_{I,I}(x,T)\;[\alpha_{2I}+I_1(x,\tau)]^2\mathrm{d}\tau\\ \qquad -\displaystyle\int\limits_0^t k_{I,V}(x,T)\;[\alpha_{2I}+I_1(x,\tau)]\,[\alpha_{2V}+V_1(x,\tau)]\;\mathrm{d}\tau\\ V_2(x,t)=\dfrac{\partial}{\partial x}\displaystyle\int\limits_0^t D_V(x,T)\dfrac{\partial\, V_1(x,\tau)}{\partial\, x}\mathrm{d}\tau+f_V(x)-\int\limits_0^t k_{V,V}(x,T)\;[\alpha_{2V}+V_1(x,\tau)]^2\mathrm{d}\tau\\ \qquad -\displaystyle\int\limits_0^t k_{I,V}(x,T)\;[\alpha_{2I}+I_1(x,\tau)]\,[\alpha_{2V}+V_1(x,\tau)]\;\mathrm{d}\tau\end{cases} \tag{3e}$$

$$\begin{cases} \Phi_{2I}(x,t) = \dfrac{\partial}{\partial x}\displaystyle\int_0^t D_{\Phi_I}(x,T)\dfrac{\partial \Phi_{1I}(x,\tau)}{\partial x}\mathrm{d}\tau + \int_0^t k_{I,I}(x,T)\,I^2(x,\tau)\,\mathrm{d}\tau \\ \qquad + \displaystyle\int_0^t k_I(x,T)\,I\,(x,\tau)\,d\tau + f_{\Phi_I}(x) \\ \Phi_{2V}(x,t) = \dfrac{\partial}{\partial x}\displaystyle\int_0^t D_{\Phi_V}(x,T)\dfrac{\partial \Phi_{1V}(x,\tau)}{\partial x}\mathrm{d}\tau + \int_0^t k_{V,V}(x,T)\,V^2(x,\tau)\,\mathrm{d}\tau + \\ \qquad + \displaystyle\int_0^t k_V(x,T)\,V\,(x,\tau)\,\mathrm{d}\tau + f_{\Phi_V}(x) \end{cases} \tag{5e}$$

Average values of the second-order approximations of considered approximations by using the following standard relation [9–11]

$$\alpha_{2\rho} = \frac{1}{\Theta L}\int_0^\Theta \int_0^L [\rho_2(x,t) - \rho_1(x,t)]\,\mathrm{d}x\,\mathrm{d}t \tag{8}$$

Substitution of the relations (1e), (3e), (5e) in the relation (8) gives us the possibility to obtain relations for required average values $\alpha_{\ 2\rho}$

$$a_{2C} = 0,\ a_{2FI} = 0,\ a_{2FV} = 0,\ \alpha_{2V} = \sqrt{\frac{(b_3+E)^2}{4\,b_4^2} - 4\left(F + \frac{\Theta\, a_3 F + \Theta^2 L\ b_1}{b_4}\right)} - \frac{b_3+E}{4\,b_4},$$

$$\alpha_{2I} = \frac{C_V - \alpha_{2V}^2 S_{VV00} - \alpha_{2V}(2\,S_{VV01} + S_{I\,V10} + \Theta\,L)\ - S_{V\,V02} - S_{I\,V11}}{S_{IV01} + \alpha_{2V} S_{IV00}},$$

where

$b_4 = \frac{1}{\Theta L}\left(S_{IV00}^2 S_{V\,V00} - S_{VV00}^2 S_{II00}\right)$, $b_3 = -\frac{S_{II00}S_{VV00}}{\Theta L}(2S_{VV01} + S_{IV10} + \Theta\,L) + \frac{S_{IV00}}{\Theta L}$ $\times\ S_{VV00}\,(S_{IV01} + 2S_{II10} + S_{IV01} + \Theta\,L) + \frac{S_{IV00}^2}{\Theta L}(2S_{VV01} + S_{IV10} + \Theta\,L) - \frac{S_{IV00}^2 S_{IV10}}{\Theta^3 L^3}$,

$b_2 = \frac{S_{II00}}{\Theta L} \times S_{VV00}(S_{VV02} + S_{IV11} + C_V) - (S_{IV10} - 2S_{VV01} + \Theta\,L)^2 + (\Theta\,L + 2S_{II10} + S_{IV01})\,S_{IV01}\,\frac{S_{VV00}}{\Theta L}$ $+ \frac{S_{IV00}}{\Theta L}(S_{IV01} + 2S_{II10} + 2S_{IV01} + \Theta\,L)\,(2S_{VV01} + \Theta\,L + S_{IV10}) - \frac{S_{IV00}^2}{\Theta L}(C_V - S_{VV02} - S_{IV11})$ $+ C_I\,\frac{S_{IV00}^2}{\Theta^2 L^2} - \frac{2\,S_{IV10}}{\Theta L} S_{IV00} S_{IV01}$, $b_1 = \frac{S_{II00}}{\Theta L}(S_{IV11} + S_{VV02} + C_V)\,(2S_{VV01} + S_{IV10} + \Theta\,L) + \frac{S_{IV01}}{\Theta L}$ $\times\ (\Theta\,L + 2S_{II10} + S_{IV01})\,(2S_{VV01} + S_{IV10} + \Theta\,L) - S_{IV10}\frac{S_{IV01}^2}{\Theta L} - \frac{S_{IV00}}{\Theta L}(3S_{IV01} + 2S_{II10} + \Theta\,L)$ $\times\ (C_V - S_{VV02} - S_{IV11}) + 2\,C_I S_{IV00} S_{IV01}$, $b_0 = \frac{S_{II00}}{\Theta L}(S_{IV00} + S_{VV02})^2 - (\Theta\,L + 2\,S_{II10} + S_{IV01})$. $\times\ (C_V - S_{VV02} - S_{IV11})\frac{S_{IV01}}{\Theta L} + 2C_I S_{IV01}^2 - \frac{S_{IV01}}{\Theta L}(C_V - S_{VV02} - S_{IV11})\,(\Theta\,L + 2\,S_{II10} + S_{IV01})$,

$C_I = \frac{\alpha_{1I}\alpha_{1V} S_{IV00} + \alpha_{1I}^2 S_{II00} - S_{II20} S_{II20} - S_{IV11}}{\Theta\,L}$, $C_V = \alpha_{1I}\alpha_{1V} S_{IV00} + \alpha_{1V}^2 S_{VV00} - S_{VV02} - S_{IV11}$

$E = \sqrt{8\,y + \Theta^2\frac{a_3^2}{a_4^2} - 4\,\Theta\,\frac{a_2}{a_4}}$, $F = \frac{\Theta\,a_2}{6\,a_4} + \sqrt[3]{\sqrt{r^2 + s^3} - r} - \sqrt[3]{\sqrt{r^2 + s^3} + r}$, $r = \frac{\Theta^3 b_2}{24\,b_4^2}$ $\times\ \left(4b_0 - \Theta\,L\frac{b_1 b_3}{b_4}\right) - \frac{\Theta^3 b_2^3}{54\,b_4^3} - \frac{b_0\Theta^3}{8b_4^2}\left(4\ b_2 - \Theta\frac{b_3^2}{b_4}\right) - L^2\frac{\Theta^4 b_1^2}{8\,b_4^2}$, $s = \Theta^2\frac{4b_0 b_4 - \Theta\,L\,b_1 b_3}{12\,b_4^2}$ $-\Theta\,b_2 / 18\,b_4$.

In the framework of the paper, we calculate the considered concentrations of dopant and radiation defects by using the second-order approximation in the framework of the method of averaging of function corrections. The approximation is usually enough good approximation to make qualitative analysis and to obtain some quantitative results. All obtained results were checked by comparison with results of numerical simulations.

Discussion

In this section, we analyzed redistribution of dopant and radiation defects during their annealing. Typical distributions of concentration of dopant in the considered multilayer structures are shown in Fig. 2 at different values of the radiation dose. Increasing value radiation dose leads to acceleration of dopant diffusion during annealing. Figure 3 shows typical distributions of charge carriers mobility at different values of radiation dose. To calculate curves from Fig. 3, we taken into account the following empirical relation between the charge carriers mobility and dopant concentration: $\mu = \mu_0\ [C_0/C(x,t)]^{1/3}$ (see, for example, [12]), where μ_0 is the mobility at small value of concentration of

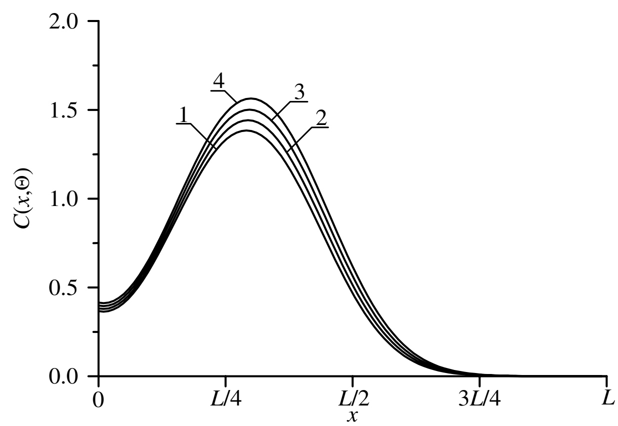

Fig. 2 Typical distributions of concentration of dopant at different values of dose of implanted ions. Increasing number of curve corresponds to decrease in value of dose of implanted ions

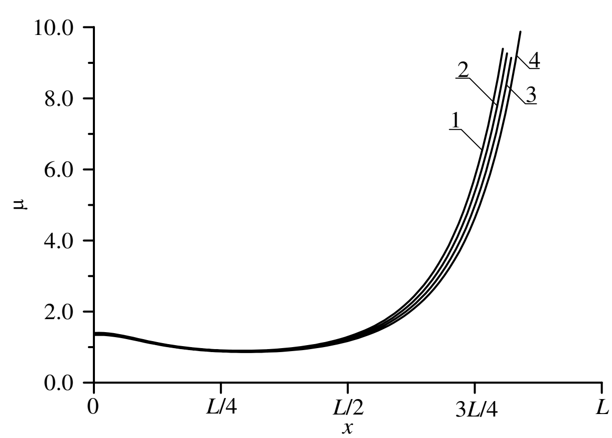

Fig. 3 Typical distributions of charge carriers mobility at different values of the dose of implanted ions. Increasing number of curve corresponds to increasing value of dose of implanted ions

dopant. Figure 3 shows that increasing the radiation dose leads to decrease in electro-physical properties of the irradiated material and decrease in the value of charge carriers mobility. At the same time, radiation processing leads to decrease in mismatch-induced stresses in multilayer structures [11].

Conclusion

In this paper, we introduce a model for prognosis radiation damage in a irradiated materials in more common case in comparison with literature. We consider an analytical approach to analyze mass transfer in more common case in comparison with literature: The approach gives a possibility to take into account nonlinearity of the considered process, as well as changes in the considered parameters in space and simultaneously in time. Based on the considered model, we determine conditions to decrease radiation damage in the irradiated materials. Also we demonstrate that radiation exposure leads to decrease mismatch-induced stresses in a multilayer structures.

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